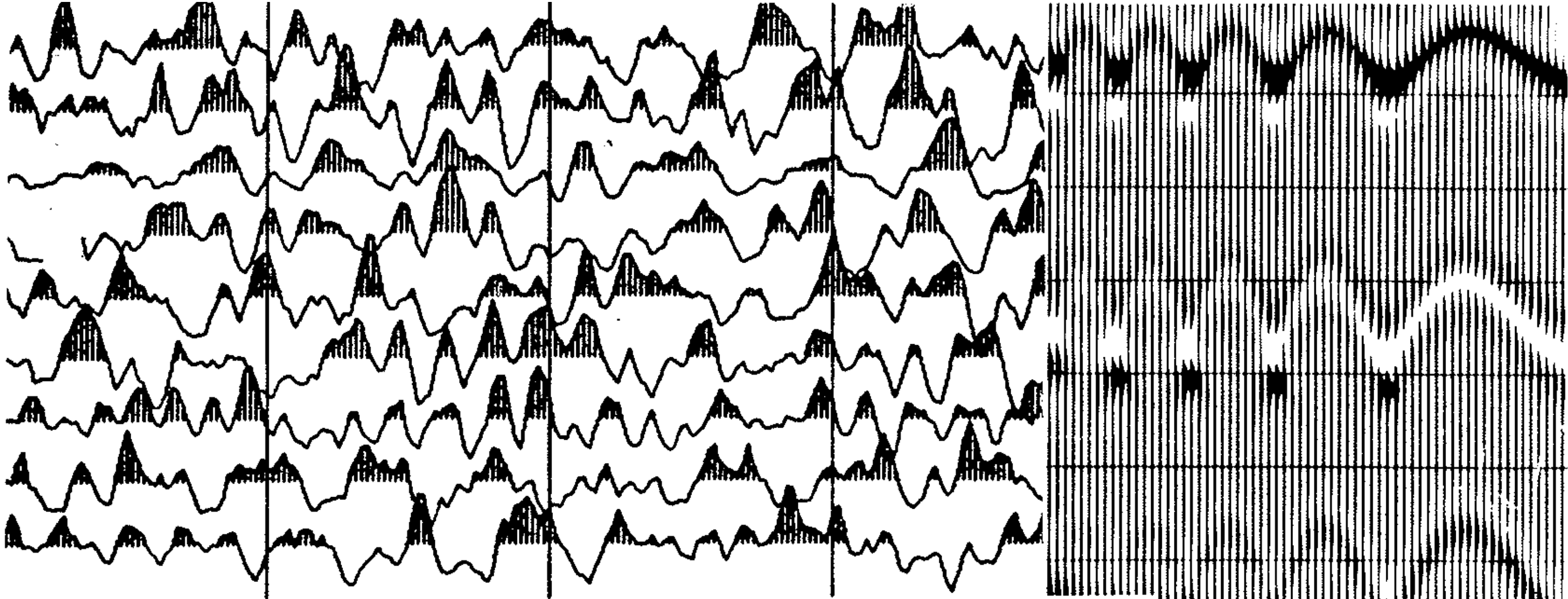
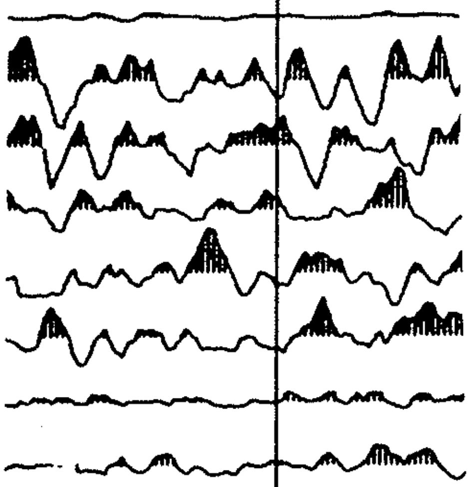
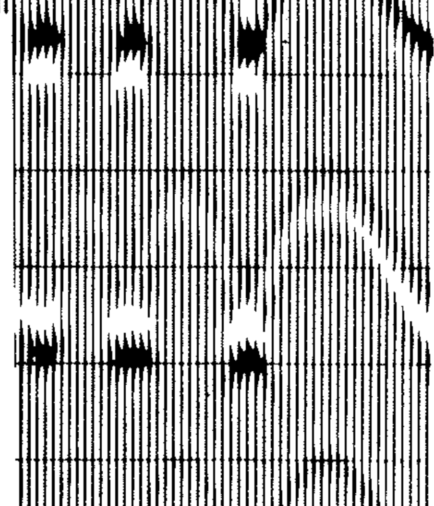
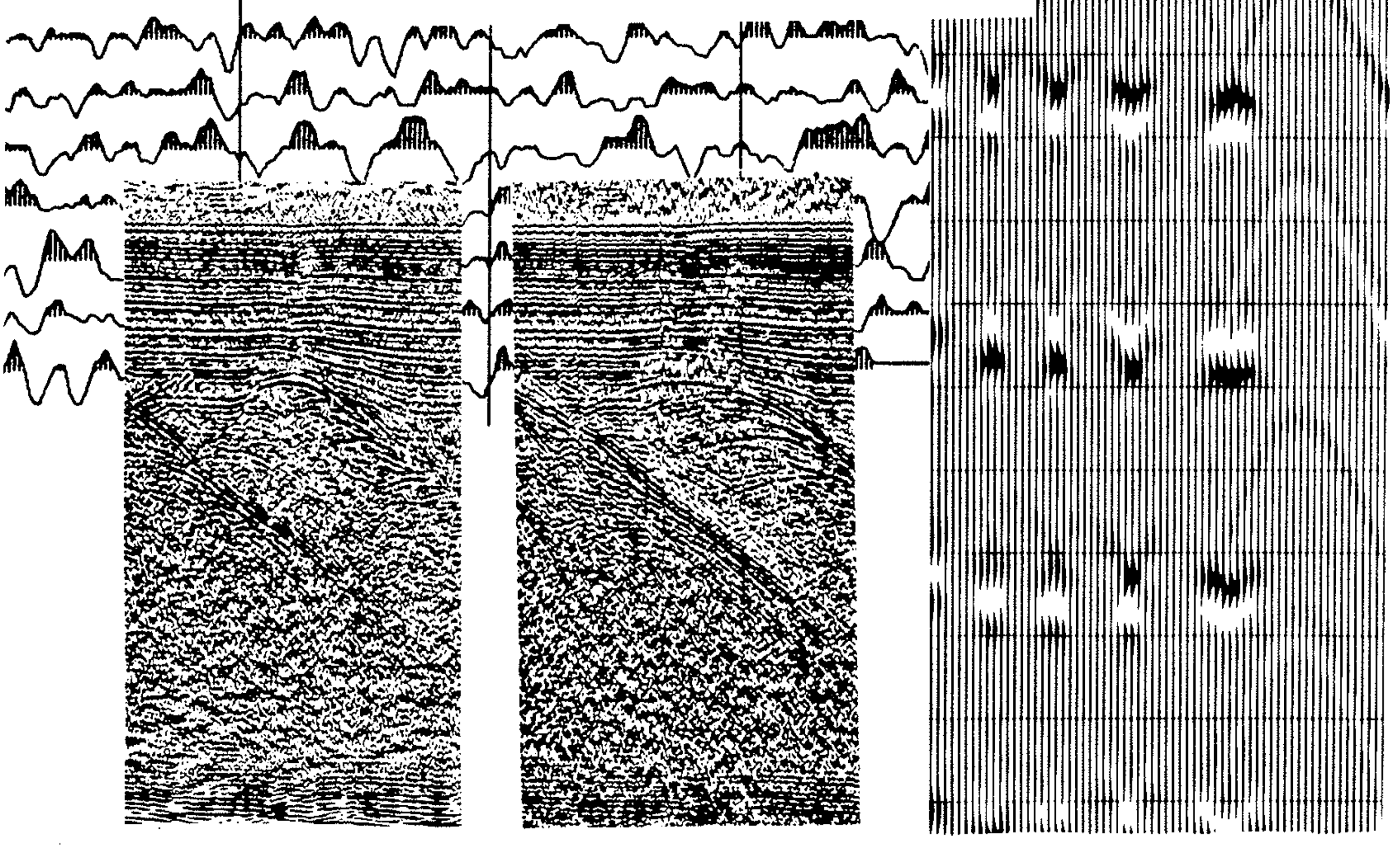




PROPOSAL TO INITIATE THE STANFORD EXPLORATION PROJECT TO DO FUNDAMENTAL RESEARCH IN REFLECTION SEISMOLOGY





A PROPOSAL TO

PETROLEUM EXPLORATION INDUSTRY FIRMS

by

Department of Geophysics
Stanford University
Stanford, California 94305

STANFORD EXPLORATION PROJECT

to do

FUNDAMENTAL RESEARCH IN REFLECTION SEISMOLOGY

Proposed Starting Date:	September 1, 1973
Project Duration:	12 months
Amount requested:	$8,000 per firm
	$80,000 total

Principal Investigator:

Jon F. Claerbout

Jon F. Claerbout
Assoc. Professor
Social Security No:
394-34-9067
(415)321-2300, x3717

Fredrick C. Kruger
Assoc. Dean for Research
School of Earth Sciences

Office of Research Administration
(Approved for the University)

ABSTRACT

It is proposed that a group of about ten petroleum industry firms pool $8,000 each in order to support fundamental research in reflection seismology at Stanford University. This research is expected to lead to improved techniques for computer interpretation of reflection seismic data in petroleum prospecting.

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FUNDAMENTAL RESEARCH IN REFLECTION SEISMOLOGY

I. Introduction

This proposal is concerned with doing fundamental research into techniques for solving the seismic wave equation in inhomogeneous media. The reason for this proposal is our belief that the correct understanding and solution of many problems in reflection seismology depend on the more rigorous and exact analysis that can be obtained by faithfully solving the wave equation. In addition, the time is appropriate for investigation of wave equation techniques since the present and easily forseeable computing power of the oil industry will make such data processing concepts practical. Because of the fundamental nature of this research, we are looking for a broad base of support from the oil industry, with the funding coming from a group of at least ten companies.

Our approach to waveform analysis is a synthesis of the disciplines of stochastic systems theory, physical optics, and finite difference numerical techniques. Despite the fact that problems of reflection seismology involve a good measure of each of these disciplines, rarely have they all been applied simultaneously. The kinds of opportunities and applications which arise from this combined approach are illustrated in an article in the October 1972 issue of GEOPHYSICS (see appendix). The article explains a method for waveform migration in velocity inhomogeneous media, using finite difference solutions to the wave equations. The method is illustrated by numerous synthetic examples and two marine sections.

Two important sub-areas of reflection seismology are now ripe both in terms of national need and in terms of the ability of technology to produce a functional solution. First, the growing difficulty of locating significant structural petroleum prospects in the continental U.S. has demonstrated the need for developing techniques for resolving subtle stratigraphic traps. Second, offshore prospecting has revealed an unexpectedly severe masking problem with the deep water multiple reflections. In both cases the primary need is more accurate simulation of seismic waveforms observed in realistically complex geologic structures. Wave equation processing techniques have already been applied to these two problems and the research done at Stanford (discussed in the next two sections of this proposal) gives promise of being able to understand and solve these problems.

The research so far has been restricted to processing techniques for conventional 2D data. The same fundamental techniques can be extended to process 3D data. However, since the computer memory requirements are much larger for 3D problems than for 2D problems, the initial investigation of 3D problems would probably be done in the frequency domain instead of the time domain.

Our fundamental studies will include theoretical work, computer testing of synthetic cases, and occasional tests with field data. Although the basic theme of this proposal is fundamental research, the wave equation techniques will be applied to real world problems. This will not only enhance the communication of ideas between the sponsors and the investigators, but will also allow realistic evaluation and direction of the research.

II. Deep Water Sea Floor Multiple Reflections

Recent surveys off Eastern Canada exhibited an unexpectedly severe masking problem with multiple reflections. Masking becomes increasingly severe with increasing water depth for two reasons. First, the group delay of a reverberation operator increases not only with reflection coefficient but also with the two way travel time, thus spreading the contaminating energy deeper into the time section. Second, the topography effect is more serious in deep water than in shallow water. A measure of focusing is found by dividing sea floor depth by a typical topographic radius of curvature. In other words, for a given topography, the focusing effect becomes more severe with increasing water depth. The focusing effect results in amplitude and waveform anomalies on the primary reflected wave which are of increasing severity on higher order multiples. This effect is not modelled by the deconvolution method and is, we believe, what Hofer, Schneider and McBeath (East Coast Canada Marine Seismic Data: Geophysical Problems and Current Approaches, presentation, 42nd annual meeting of SEG) referred to as "geometrical deconvolution detuning". We have learned how to model this effect and as a part of the proposed research we would like to investigate whether such modelling can form the basis for a means to alleviate the masking problems.

A procedure which comes to mind is to take the observed primary, along with all its amplitude anomalies and diffractions, and compute the expected multiple waves, then adaptively subtract the anticipated multiples from the actual ones. This could proceed in a fashion somewhat like that which we illustrated in "Time and Space Adaptive Deconvolution Filters", a presentation by Don C. Riley and John Parker Burg at the 1972 SEG meeting. Since adaptive methods are fairly well known the rest of this section will show only the unpublished results we have already attained on synthesis of sea floor multiple reflections.

We have solved the acoustic wave equation in normal moveout coordinates. In obtaining the differential equation governing multiples, we made the Fresnel approximation and the thin lens approximation. (When conditions warrant both approximations can be rather easily improved.) The partial differential equation is also set up to simulate recording with programmed gain control so that all orders of multiples would come out with the same amplitude if the sea floor were flat. Figure 1 shows the synthetic time section. Early in the time section it is easy to recognize that the dip on the N-th order multiple is nearly equal to N times the dip of the primary. Also focusing can be perceived where the multiples sag downward. Late in the section the situation becomes highly focused and confused. That these kinds of phenomena do exist on field data is clearly exhibited in Figure 2. Even in the shallow water of the Chukchi Sea the focusing effects of sea floor topography on multiples are very pronounced. It will be observed that an irregular sea floor (probably resulting from differential glacial erosion of the Mio-Pliocene fold structure) results in weaker high order multiples where the sea floor is locally convex. Through this "window" the AGC brings the steeply dipping structure into view. These phenomena are most clearly understood on near trace sections. They have been unrecognizable to us on the stacked sections we have seen. We have not yet seen any near trace sections from Eastern Canada and hence are unable to assess quantitatively the comparative importance of sea floor topography in the prediction of those multiples. However, the promise of practical results arising from fundamental studies seems quite clear.

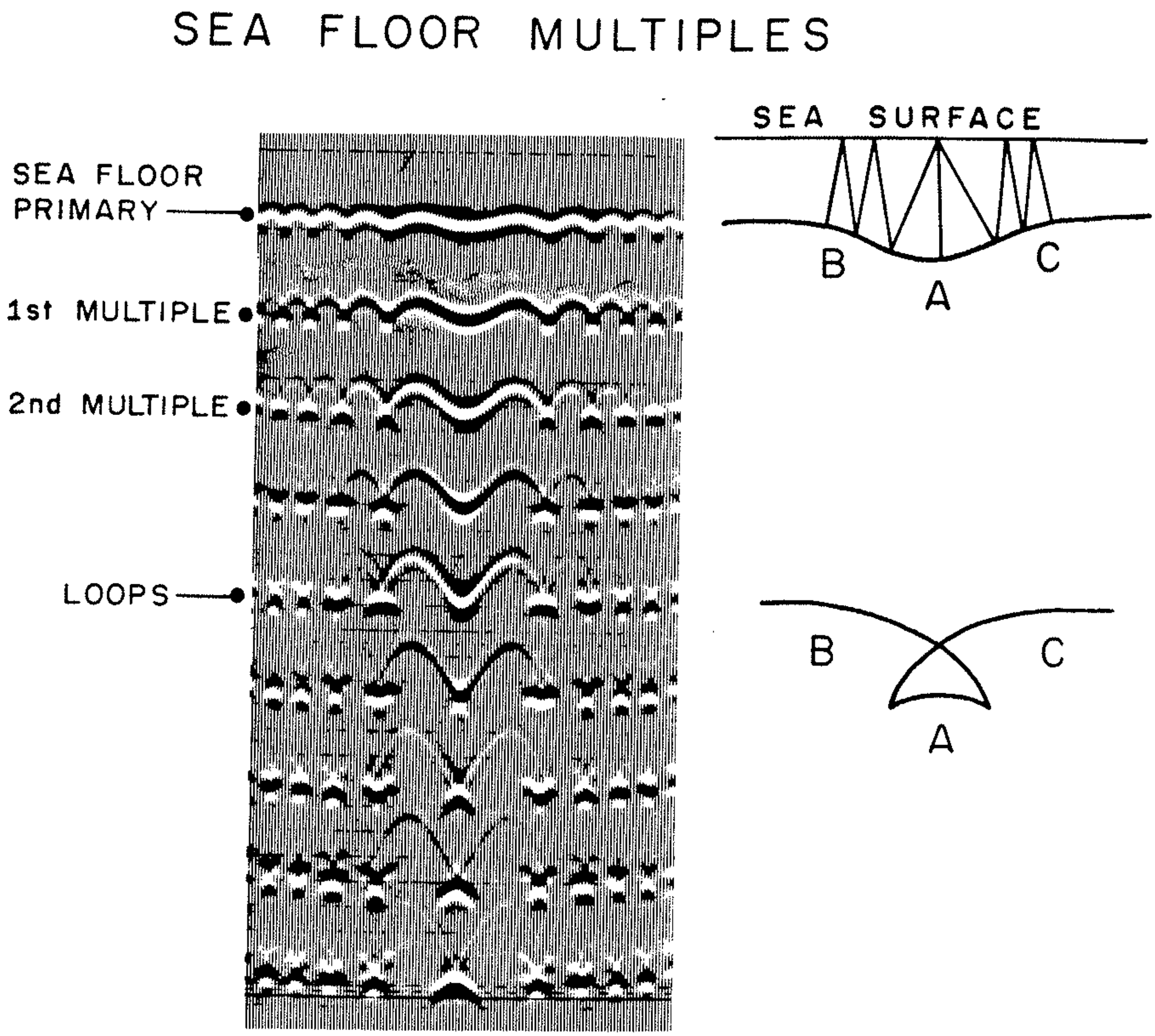

Figure 1. Synthetic Sea Floor Multiple Reflections (right) and ray diagram (left) to explain travel time loops. Note increasing dip on higher order multiples and dominance of focusing effects at late times. Note 90° phase shift of focused waves.

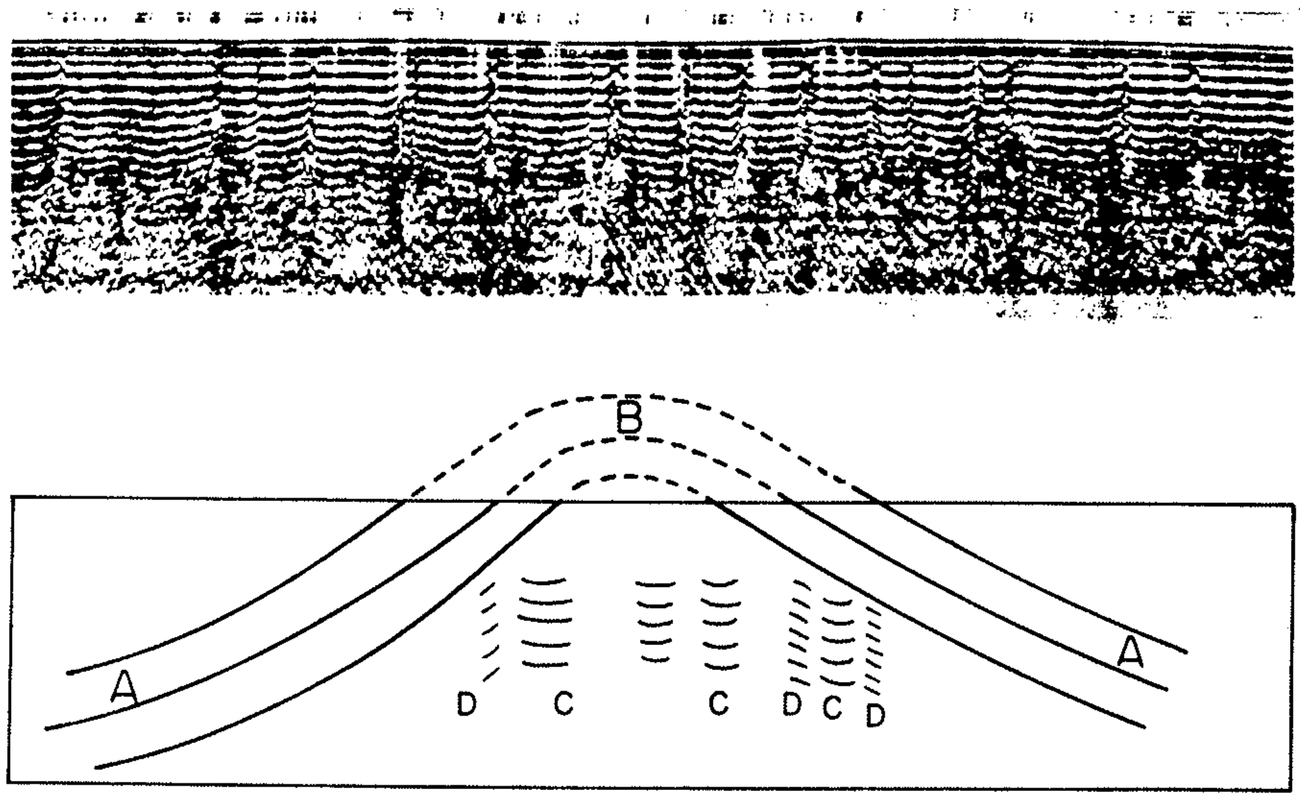

Figure 2. Example of focusing effects on multiple reflections in near trace section at Chukchi Sea. These effects are obscured by stacking.

A. Existing structure

B. Former structure unevenly eroded away leaving localities of sea floor convex or concave

C. High order multiple reflections focusing where the sea floor is concave.

D. Existing structural dip exposed in windows where the multiples are weak (i.e., where convex sea floor causes multiple to spread rapidly).

A part of the computer program which created Figure 1 can be run backwards. This "running backwards" of the program which creates synthetic multiples from a known sea floor topography we will choose to call "multiple migration" because of the analogy with primaries. The center frame of Figure 3 is the same synthetic data as Figure 1. The multiple migration of this data is shown on the right frame of Figure 3. The highly confused pattern of multiples has become flat lying and quite regular. This process of organizing and regularizing multiples can be expected to be extremely useful in their adaptive elimination. If the multiples are migrated under the incorrect assumption that they are primaries the left part of Figure 3 results. Basically the strong lateral amplitude variations on the time section give rise to circular arcs (there is a 5:1 vertical exaggeration) in the migration. We have seen such arcs on migrated data. Perhaps they can be used as another means of distinguishing multiples from primaries.

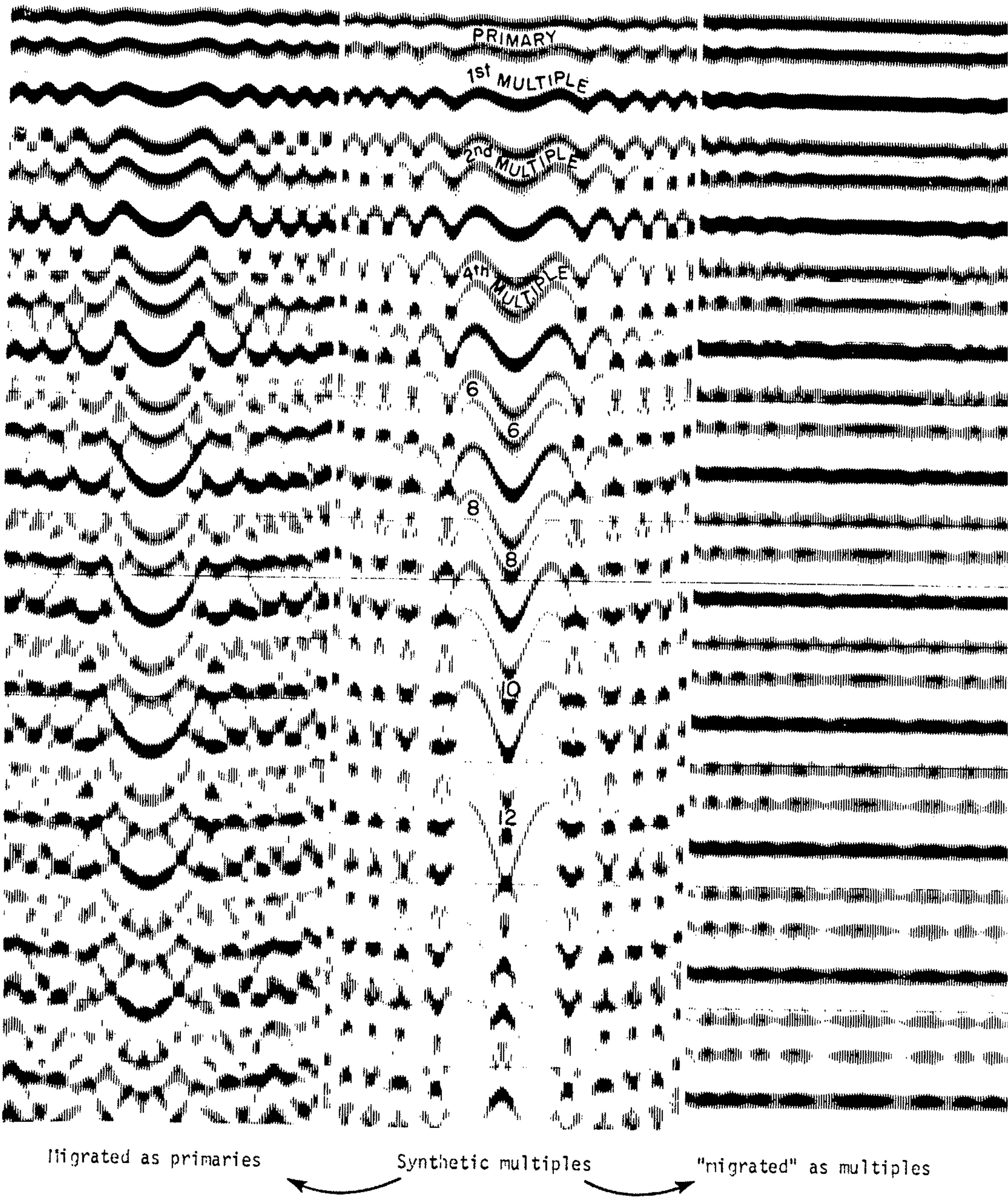

Figure 3. "Migration" of Synthetic Multiples

III. Resolution of Stratigraphic Traps

The reflection seismic method is well suited for defining structural prospects since it is relatively easy to detect horizontal and vertical velocity discontinuities. However, stratigraphic traps are often indicated by velocity gradations rather than discontinuities. Reflection seismology must be pushed to the extremes of its capabilities in order to get a suitable resolution of these gradations. None-the-less we find that computer model building techniques in reflection seismology are extremely crude compared to those in other branches of geophysics. Because of this we believe there is at present a considerable opportunity for theoretical investigation. At the present time reflection seismic velocity analysis is based on "picks" of events in the space of time, NMO, and dip (Sherwood and Poe, "Continuous Velocity Estimation and Seismic Wavelet Processing", GEOPHYSICS, Oct. 1972). Some idea of the kind of information carried in waveforms but probably not carried nor readily extracted from picks is indicated in Figure 4.

It is our believe that the maximum amount of information will be extracted from the data only when modern "inversion" methods are applied to the whole wave field. (The word "inversion" comes about because the standard text book problem, or forward problem, is the calculation of waves given the complete specification of the material and geometry whereas in the inverse problem one starts with the waves and deduces the material and geometry. Also in inverse geophysical problems one usually ends up inverting a lot of matrices.) The matrix involved in the reflection seismic problem would be called the partial derivative matrix of the theoretical seismograms with respect to the model parameters (e.g.

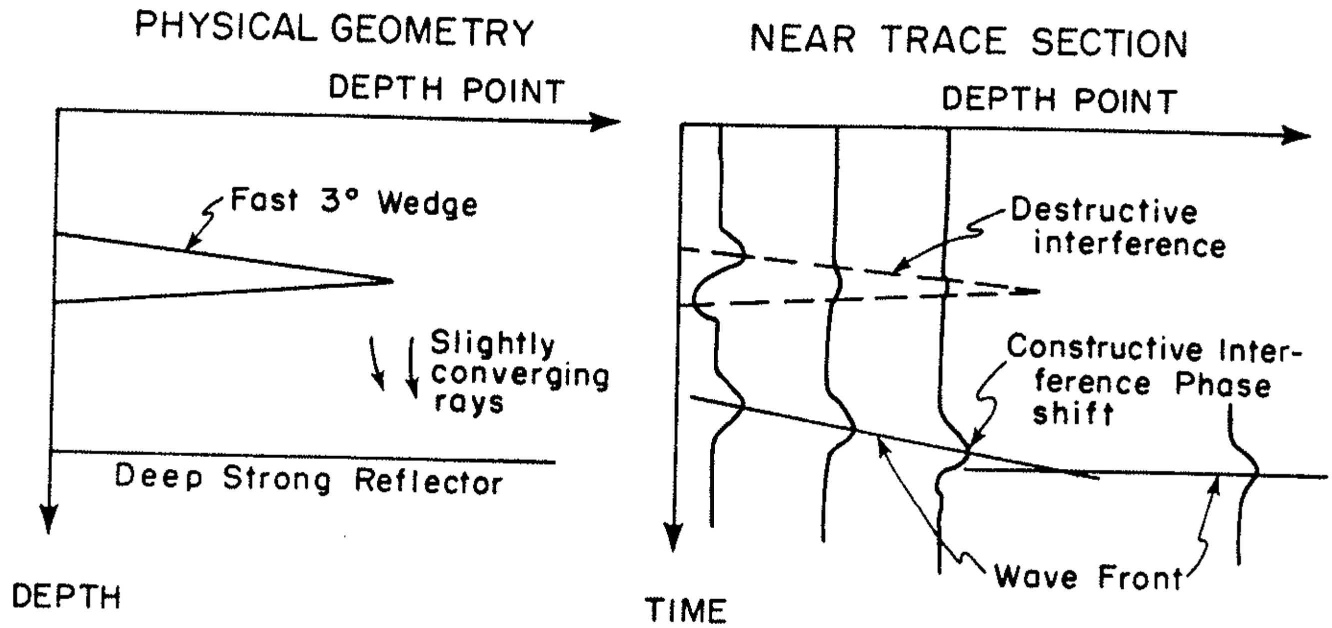

Figure 4. Illustration of a simple stratigraphic trap geometry where the existence of a pinchout is exhibited seismically by amplitude and waveform anomalies some distance below the pinchout. It would be worthwhile to compute synthetic data of this nature and see how well the tip of the pinchout can be located. If it is theoretically locatable then it would be worthwhile to attempt to identify this situation with field data examples, and attempt to find a theoretical method which would make systematic and optimal use of these wave phenomena in the construction of a velocity model. The limitations on this method will be different from the 5/8 wavelength resolution limit commonly assumed. However, we don't know if they will be better or worse.

velocity as a function of position). Since a mile of seismic survey collects about 2 million words of seismic data and if we take the information density as perhaps as much as 100 times less than that of the data, then it is clear that the partial derivative matrix is immense (about 10^{10} words). Furthermore we must recompute and solve the matrix many times as we iterate to a solution. In addition local minima in the error surface can present problems.

Clearly, a straight-forward approach applying the method of say Backus and Gilbert ("Numerical applications of a formalism for geophysical inverse problems", GEOPHYS. J. ROY. ASTR. SOC., 1967, v. 13, p. 247-276) directly to the seismic waveforms is impractical. However, even in their application (they determine velocity as a function of depth only) they do not work directly with all the waveforms but instead extract only a modest number of free oscillation frequencies as "data" for the inversion. In other words what we are saying is that our partial derivative matrices are actually not very sparse but if we are clever we may be able to find a suitable set of approximations and transformations to transform them to sparse enough matrices that we can afford to deal with them. We are proposing theoretical research which would attempt to determine a practical solution of the problem of deducing an accurate stratigraphic-trap-like velocity structure from the observed waveforms. Because of our previous experience in computing numerical solutions to the wave equation we can at least compute portions of the partial derivative matrix.

Inversion methods are more highly developed in potential and diffusion theory than in seismology. Because they generally have a simpler physical situation and far less data, people working in these fields have extensively used the partial derivative matrix method of inversion. They routinely determine 2 and 3 dimensionally inhomogeneous models for things like resistivity. It is entirely probable that seismologists could benefit from a thorough grasp of the material contained in Theodore R. Madden's work "Transmission Systems and Network Analogies to Geophysical Forward and Inverse Problems" (Office of Naval Research Report N-0001-14-67-A-0204-0045/371-401/05-01-71). Fortunately Prof. Madden has agreed to visit Stanford for 4 months and would participate in the proposed research.

IV. A Central Problem in Seismic Waveform Analysis

We wish to state an extremely important unsolved inverse problem in seismology in simplest form stripped of all practical complications. It will be apparent that this problem is related to the problem of resolving stratigraphic traps.

Consider a transient time function being carried by a plane wave. The wave enters a region of weak 2D or 3D velocity inhomogeneity and propagates on through it to the other side where it is observed. The observations are distorted wave forms which now are variable along the wavefront. Tracking this wave either forward or backward in time and space is a fairly simple procedure with our difference methods provided that the velocity inhomogeneity is known. An example of something close to this is attached (Fig. 5 from p. 302 of Lecture Notes of Jon Claerbout). It is described in greater detail in "Extrapolation of Time-Dependent Waveforms along their Path of Propagation" by Jon F. Claerbout and Ansel G. Johnson, GEOPHYS. J. ROY. ASTR. SOC., 1971, v. 26, p. 285-293, which is also attached as an appendix. The inverse problem, which we believe to be important, is to deduce the material velocity variation required so that as the laterally variable observed wave is projected backwards through the inhomogeneity it becomes increasingly simpler until we get to the point of entry where it should turn into a simple transient waveform carried by a plane wave. We regarded this inverse problem as hopelessly difficult until very recently when we made some theoretical progress on it. The basic idea will be briefly indicated.

View frames labeled t_0, t_1, t_2, etc. in the figure as representing a

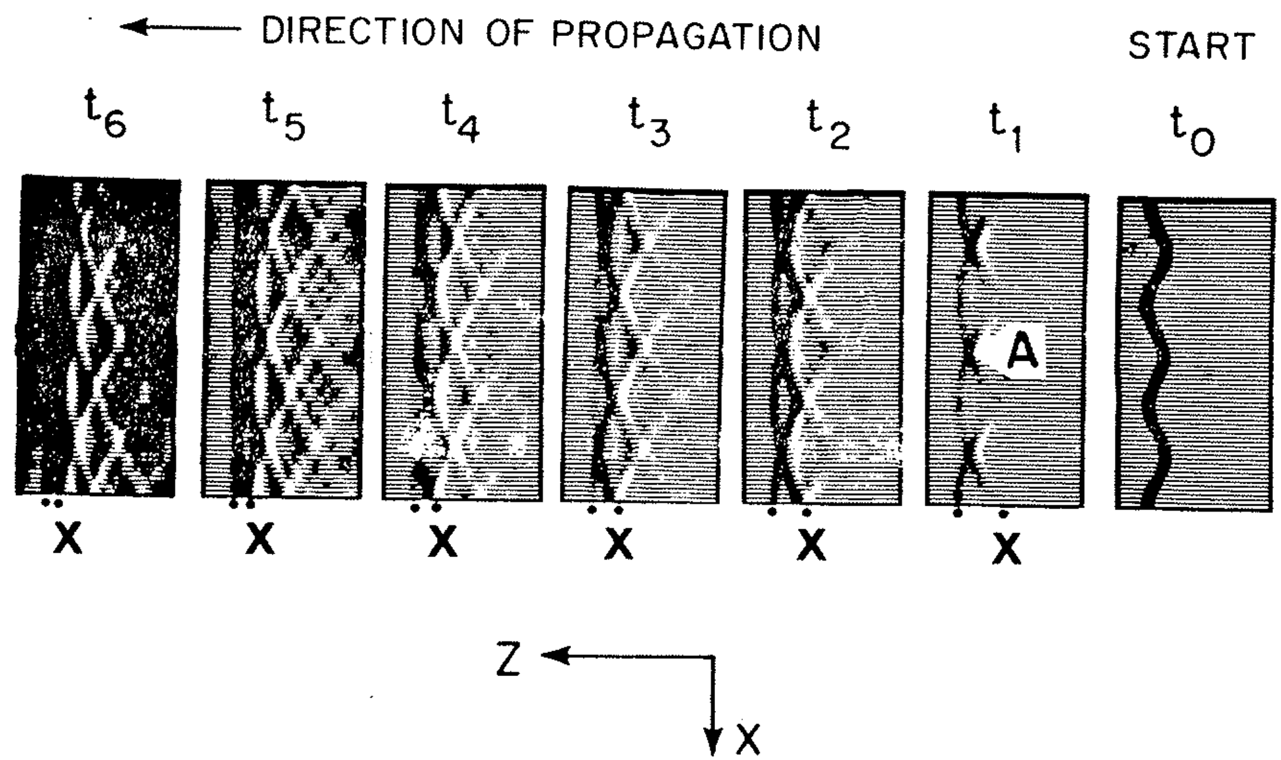

Figure 5. Disturbed plane wave propagating through a homogeneous medium. The first arrival of a disturbed plane wave heals itself during propagation. The wave coda or trail gets more and more complicated and energetic. In the trail, energy moves back away from the first arrival while phase fronts (marked by "X" move forward. Beam-steer signal processing (sum over the x-coordinate) enhances the first arriving signal but trys to destroy later arriving signals (the trail).

wave pressure field $P(x,t)$ observed at successive values of depth z_0, z_1, z_2, etc. (To the Fresnel approximation z and t are interchangeable.) Now notice that in the frame labeled t_0 an operation like "statics correction" could remove all the x variation of pressure from the frame. Next notice that the same is not true for frames t_2 through t_6. Now recall that the waves displayed actually encountered velocity inhomogeneity only right at or just before frame t_0. From these observations it is possible to assemble a "dynamic programming" type of estimation of the velocity inhomogeneity. When waves are back projected from z_j to z_{j-1} the velocity that is estimated should be such that a chosen functional of the wave field, $P(x,z_{j-1},t)$ is minimized. For instance, as a starting point, one could estimate the velocity which produces a $P(x,z_{j-1},t)$ which has the minimum x variation. Trial of this criteria shows that it is not completely satisfactory. However, it is our feeling that several more sophisticated criteria that may work in the regions where this simple idea failed should be investigated.

V. Administrative Provisions

1. Meetings with sponsors.

There would be an annual meeting with sponsors at the location of the Society of Exploration Geophysics meeting during or immediately following the convention. (Both researchers and sponsors would ordinarily be attending this annual Monday/Thursday meeting anyway.) The budget for the first year of the research provides for two meetings. The first is about a month after the initiation of the project (October 26, 1973); the second meeting would be a year later, about a month after the completion of the first year's efforts. The purpose of the first meeting would be to clarify general organization and matters such as the providing of data and computing services. The purpose of the second meeting would be to report results.

2. Reporting

There would be a semi-annual and annual technical report, and an annual financial report. Sponsors would be sent preprints of papers, M.S. theses, Ph.D. theses, etc. Ultimately, research results would generally appear as journal articles. Sponsors would be suitably acknowledged. The semi-annual reports would serve the purpose of keeping sponsors up to date on our progress. The semi-annual reports would also include detailed material of a nature which is not especially suitable for publication such as computer programs, the methods and examples which turned out unsuccessful, detailed algebraic derivations, etc.

3. Funding

The project will begin September 1, 1973. The budget section is based on 10 contributors at $8,000 each. However, if there are at least 5 contributors the the project will begin at a reduced level of activity.

4. Reallocation

The principal investigator would have authority to reallocate funds among budget categories. The sponsors could, on request, receive monthly budget statements.

5. Redirection

The principal investigator could introduce additional topics for research not mentioned in this proposal if they fall clearlv in the realm of fundamental research in reflection seismology.

6. Consulting

Sponsors could visit the project at any time to be informed of progress. Participating firms may request special lectures or discussions on the work of the project at a mutually satisfactory time. The requesting company would pay expenses and an honorarium. The principal investigator and any other full time investigator would not enter into confidential consulting arrangements in this field of research during the period of sponsorship.

7. Patents

All patents coming out of work by the STANFORD EXPLORATION PROJECT will be owned according to the Stanford Universtiy patent policies. However, all participating firms will be given royalty-free use of the patented information.

8. Continuation

This work could be renewed for subsequent years as long as a sufficient number of sponsors remained interested in the research.

9. Procurement of data and additional computer services

Data would be procured informally from sponsors either without charge or for the costs of magnetic tapes and mailing. Additional computer services would also be solicited from sponsors. We can use about 100 hours per year of sponsors' overcapacity second and third shift time. (In the unlikely event that all of our sponsors are utilizing 100% of their capacity our budget at the Stanford Computation Center is sufficient for the proposed research although evaluations would be substantially curtailed.) We would receive these services either without charge or for only the real incremental operating costs such as paper, film, mailing, telecommunications, etc.

VI. BUDGET

September 1, 1973 to August 31, 1974

A. SALARIES	
Principal Investigator - Jon F. Claerbout	
30% academic year @ $1778/month	$4,801
2 months summer	3,556
Research Associates:	
John P. Burg	
60% time 12 months @ $1,500/month	10,800
Theodore R. Madden	
50% time 4 months @ $2,100/month	4,200
Graduate Student Research Assistants:	
50% academic year @ $560/month	
Stephen M. Doherty	2,520
Robert H. Brune	2,520
Don C. Riley	2,520
Secretarial help, 50% time 12 months @ $630/month	3,780
	$34,697
B. STAFF BENEFITS 17.3% of salaries	6,003
C. TRAVEL	
1973 meeting with Sponsors	2,000
1974 meeting with Sponsors	1,500
	$3,500
D. SUPPLIES, MATERIALS & SERVICES	
Photography and reproduction	600
Office supplies and telephone	350
Data transmission over phone lines	3,680
	$4,630
E. STANFORD COMPUTATION CENTER	
Terminal rental IBM 2741, 50% @ $140/month	840
IBM time	7,868
	$8,708
F. INDIRECT COSTS	
46% total direct costs (A + B + C + D)	22,462
TOTAL PROJECT COST	$ 80,000

VII. Personnel

John Parker Burg received a B.S. in Physics and a B.A. in Mathematics from the University of Texas in 1953 and an M.S. in Physics from M.I.T. in 1960. Burg was employed by Texas Instruments Inc. from June 1956 to January 1973. He began in the Apparatus Division working on infrared systems and in 1960 was transferred to the Science Services Division to work on an AFTAC contract for the detection of underground nuclear explosions. His development of processing and analysis techniques for seismometer array data has been the basis of many present day digital oil exploration techniques. His Geophysics paper on three-dimensional frequency-wavenumber filtering has been highly referenced and in 1967 he was given the SEG best technical presentation award for the paper "Maximum Entropy Spectral Analysis". As a senior Research Geophysicist in charge of the Basic Research Section of the Science Services Research Laboratory, Burg was also responsible for supporting the anti-submarine warfare research of the division. In 1969, Burg left to attend Stanford University where he is presently working on his Ph.D. thesis in geophysics. Burg holds the patent on the well-known ARD deconvolution process and is the inventor of the Burg technique of spectral analysis. He is a member of the SEG and a senior member of the IEEE.

John Parker Burg
O6

Jon F. Claerbout
R11, R23

Jon F. Claerbout received a B.S. degree in Physics from M.I.T. in 1960, where, for a thesis, he constructed a rubidium vapor magnetometer. In 1963 he received an M.S. degree in Geophysics from M.I.T. with a thesis on seismic data processing. He worked for Teledyne, Inc. and spent a year at Uppsala University in Sweden working on multiple time series analysis. In 1967 he received a Ph.D. degree in Geophysics from M.I.T. for a thesis entitled "Electromagnetic Effects of Atmospheric Gravity Waves". A member of SEG, he is an associate professor of geophysics at Stanford University.

Don C. Riley received a B.S. degree in Geophysics from the University of Tulsa in 1970. He is presently attending Stanford where he is working on a Ph.D. in Geophysics. During the summers of 1969-70 he was employed by Texaco Inc. (Tulsa division) interpreting sparker profiles. During the summer of 1972 he was employed by the U.S. Geological Survey developing systems for processing sparker data and assisting on the recent cruise gathering data in the Chukchi-Beaufort Sea. Riley is a member of Sigman Pi Sigma, AGU, and the SEG.

Don C. Riley
O6, R23

Doherty

Stephen M. Doherty was born in New York city in 1948. He received his B.A. degree in physics from the University of Colorado in 1970 and is presently a graduate student and research assistant at the Department of Geophysics, Stanford University. His current research interest is the development of a modeling technique for the interpretation of refraction seismograms based on finite-difference solutions of the wave equation.

Theodore R. Madden is professor of geophysics at M.I.T. He did important early work in the use of induced polarization as a method of copper exploration.

Robert H. Brune attended the University of Missouri at Rolla from 1966 to 1969, receiving a B.S. in Geophysics. During the summer of 1968 he was employed by Pan American Petroleum Corp. working in reflection seismic processing. He was employed by Geophysical Services Inc. for 3 years in land and marine seismic exploration and in processing and interpreting marine gravity and magnetic data. Presently he is a graduate student in geophysics at Stanford University. He is a member of SEG, EAEG, and AGU.

BIOGRAPHICAL SKETCH OF T. R. MADDEN

Education

M.I.T., 1942-43, 46-49; graduated with B.S. in Physics
M.I.T., 1949-54, 1961; graduated with Ph.D. in Geophysics

Professional Experience

Lamont Geological Observatory, Columbia University; geophysicist on submarine geophysical expeditions summer of 1948, 49, 50 and 52

Phelps Dodge Corporation; geophysicist in mineral exploration work summer 1951

Bear Creek Mining Company (Kennecott); geophysical exploration summer of 1954, 55, and 56, 57

M.I.T., Instructor, 1951-54, Lecturer 1954-57, Assistant Professor, 1957-62, Associate Professor, 1962, Professor, 1967, M.I.T., Department of Earth and Planetary Sciences.

Visiting Associate Professor UCSD 1963-64

Consultant

Geoscience Inc.

VIII. Bibliography of Jon F. Claerbout

Digital filters and applications to seismic detection and discrimination, 1963, M.S. Thesis at M.I.T.

The error in least-squares inverse filtering, Geophysics, v. 29(1), (with E.A. Robinson), 1964

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Spectral factorization of multiple time series, Biometrica, v. 53, 1966

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Jet stream associated gravity waves and implications concerning jet stream stability, (with T.R. Madden), Atmospheric Gravity Wave Sym. Proc., July 1968. U. S. Govt. Printing Office

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Synthesis of a layered medium from its acoustic transmission response, Geophysics, v. 33(2), p. 264, 1968

Effects of thin soft layers on body wave seismograms, (with Tom Landers), Bull. Seism. Soc. Amer., v. 59(5), 2071-2078, 1969

Components as non-unique functions of total magnetic field; a note on extra-terrestrial magnetic prospecting, J. Geop. Res., v. 74(16), p. 4188, 1969

Coarse grid calculations of waves in inhomogeneous media with application to delineation of complicated seismic structure, Geophysics, v. 35(3), 1970

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Bibliography of T. R. Madden

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1960 Preliminary report on crustal magnetotelluric measurements, J.G.R., v. 65, 4204-4205, (with T. Cantwell)

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1968 Jet steam associated gravity waves and implications concerning jet steam stability, in Acoustic-Gravity Waves in the Atmosphere, ibid, p. 121-134

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TECHNICAL APPENDICES

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Extrapolation of Time-Dependent Waveforms along their Path of Propagation

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(Received 1971 July 1)

Summary

Time-dependent waveforms are commonly extrapolated in space by means of rays and occasionally by means of diffraction integrals. It is possible to extrapolate time-dependent waves in space with a partial differential equation derived from the wave equation. There are stable numerical approximations. An example illustrates a mechanism for 'signal-generated noise' which is consistent with observations.

1. Introduction

When a wave propagates in an inhomogeneous medium the waveform changes. Given that the wave has been observed at a suitable number of points in space we may attempt to solve two types of problems. First we may attempt to ascertain the nature of material inhomogeneity along the wave paths, and second, we may attempt to extrapolate the disturbance back to the source in an attempt to discover the nature of the source. With few exceptions, the methods used during the past decade for doing this kind of geophysical work may be summarized as follows: when wave equations are to be used, separability is achieved by considering cases in which the material inhomogeneity is a function of only one spatial co-ordinate. When higher-dimensional inhomogeneity is so severe that it cannot be ignored, then the wave equations are almost always specialized to ray theory. Ray theory is especially useful when only the travel time is required. Although the amplitude may also be obtained by ray theory it is often of marginal utility because amplitude measurement is made ambiguous by changing waveforms. What we develop in this paper is a finite difference approach to the wave equation which tracks the time dependent waveform of a travelling wave in two-dimensionally inhomogeneous material. This is an extension of earlier work done by one of the authors in the frequency domain. Although the Fourier transform relates time-domain solutions to frequency-domain solutions, there are several compelling practical factors which give impetus to this study. When a waveform is small at certain times of interest and large at times which are not of interest, then a satisfactory approximation to the Fourier integral may be difficult to obtain even if values are obtained with good accuracy at many frequencies: an example is the head wave. Another example occurs in reflection seismology where the most interesting part of the waveform is the late-arriving weak echoes. Another example is when the time function is of long duration but only a small portion of it is of interest; this is usually the case with short-period earthquake seismograms where there is never any hope of interpreting more than the wave packets which come from identifiable phases.

Waves move quickly into a large volume of space. Given that it seems to require about 10 sample points per wavelength to achieve even modest computational accuracy and that a typical computer memory contains 100 000 memory cells, it is evident that some kind of practical limit is attained when a wave emitted by a point source in two dimensions has expanded to a radius of 15 wavelengths. This is grossly inadequate for most geophysical examples of air waves, water waves, and seismic waves. Simplification can often be achieved by not attempting to describe the entire volume V, but only a reduced volume V_1 which surrounds the path from the source to the receiver. If a reduced volume V_1 is to be used, care must be taken to avoid artificial reflections from the sides of the volume V_1. Further economy may be achieved if an even smaller volume, say V_2, moves with a wave packet along a wave path.

2. The differential equation

Let us begin the analytical discussion by transforming the scalar wave equation

$$0 = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} \tag{1}$$

into a co-ordinate frame which translates along the z-axis at the wave speed c. For simplicity in discussion we will make various restricting assumptions. Specialists will recognize that most of these assumptions can be relaxed at the cost of a more complicated development. First, we have obviously chosen the z-axis as being along the path of interest. Second, we will neglect gradients of velocity c while retaining the space variation of c. This is a high-frequency approximation.

Energy which propagates with a component in the positive z-direction in a fixed frame will remain stationary or fall backwards with respect to a frame which translates along z at speed c. Let us choose for the co-ordinate transformation

$$\left.\begin{aligned} x' &= x \\ z' &= ct - z \\ t' &= t \end{aligned}\right\} \tag{2}$$

(see Fig. 1). Since we have chosen z' to be directed opposite to z we will have energy moving with a positive velocity component in either co-ordinate frame. Let P' denote the disturbance in the moving frame. We have

$$P(x,z,t) = P'(x',z',t'). \tag{3}$$

It will be convenient to use subscripts to denote partial derivatives. Obviously $P_x = P'_{x'}$ and

$$P_{xx} = P'_{x'x'}. \tag{4}$$

Also

$$P_z = P'_{x'}x'_z + P'_{z'}z'_z + P'_{t'}t'_z = -P'_{z'}$$

so

$$P_{zz} = P'_{z'z'} \tag{5}$$

and

$$P_t = P'_{x'}X'_t + P'_{z'}z'_t + P'_{t'}t'_t = cP'_{z'} + P'_{t'}$$

so

$$\begin{aligned} P_{tt} &= c(cP'_{z'z'} + P'_{t'z'}) + cP'_{z't'} + P'_{t't'} \\ &= c^2 P'_{z'z'} + 2c\,P'_{z't'} + P'_{t't'}. \end{aligned} \tag{6}$$

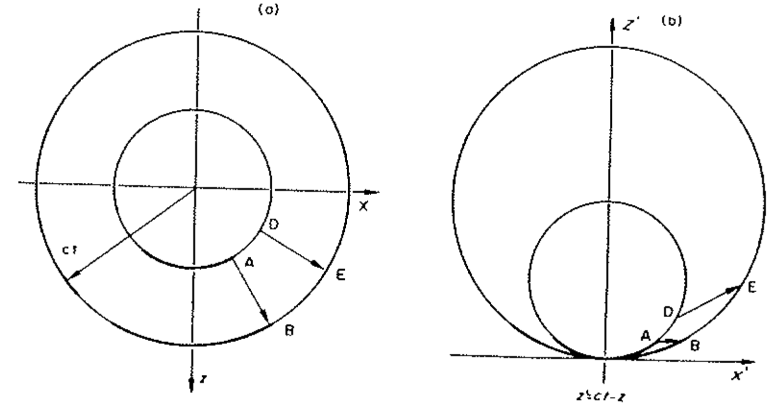

Fig. 1. Expanding spherical wave in fixed co-ordinates (left) and in co-ordinates which translate in the z-direction with the velocity of the wave (right).

Now we may insert (4), (5) and (6) into (1) to obtain

$$0 = P'_{x'x'} - 2c^{-1}P'_{z't'} - c^{-2}P'_{t't'}. \tag{7}$$

Our main interest is with those waves which propagate with approximately the velocity of the new co-ordinate frame. In the moving frame such waves are Doppler-shifted near to zero frequency. This suggests omitting the $P'_{t't'}$ term from (7). Thus (7) becomes

$$P'_{z't'} = (c/2)P'_{x'x'}. \tag{8}$$

If $\partial/\partial_{t'}$ is replaced by $-i\omega$ then (8) is said to be in the frequency domain. This equation has been studied extensively in the frequency domain by one of the authors (JFC) in a number of earlier papers. Claerbout (1970a) considers the important details relating to computer implementation; Claerbout (1970b) shows the validity of the approximation (8) to be in the range of 20 degrees from the z-axis, and gives a method to extend the range to 90° in concept or 45° at the same computational cost as (8). Claerbout (1971) pays closer attention to space variation of the velocity $c(x,z)$. In this paper we develop a solution directly in the time domain.

To solve equation (8) in a computer the disturbance is defined initially in the $x'-z'$ plane and then time is augmented in steps of $\Delta t'$. A formulation more closely related to observations would be to assume the initial disturbance was measured as a function of x and t and then extrapolate the result along the wave path in the z direction. To achieve this we introduce the change of variables

$$x'' = x \tag{9a}$$

$$z'' = z \tag{9b}$$

$$t'' = t - z/c. \tag{9c}$$

(see Fig. 2).

The new co-ordinate frame stays fixed in space relative to the old one, but time is function of position in the new frame (not unlike the difference between universal time G.M.T. and local solar time). When an observer moves in the $+z = +z''$ direction (west) time will seem to go slower. If he moves at velocity c then time stands

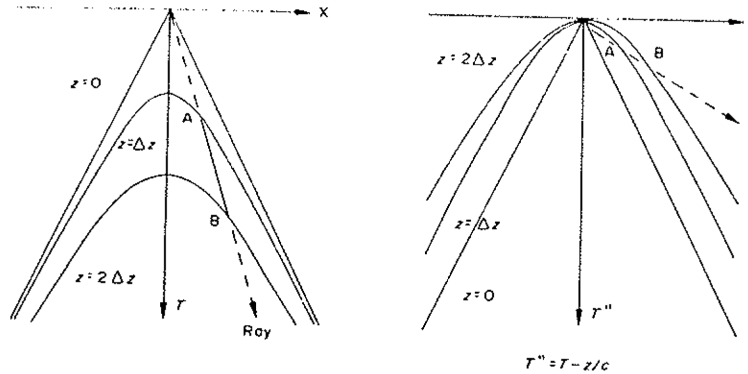

FIG. 2. A point source at $x = 0$, $z = 0$, $t = 0$. Hyperbolas at left indicate arrival times t at $z = 0$, Δz, and $2\Delta z$. When time is a function of position given by $t'' = t - z/c$ the arrival times t'' are as indicated on the right. Energy moves in the direction $+t''$, since on a wave front $z = ct\cos\theta$ and we have

$$t'' = t - z/c = t(1-\cos\theta).$$

still. Referencing time with respect to the time of the earliest possible ray is a great computational convenience because it means that a wave at the source with onset at $t_0 = 0$ will at some distance $z_1 = z_1''$ have its onset at $t_1 = z_1/c$ in the old frame but in the new frame the onset is still at $t_0'' = 0$. This means the wave onset does not move off the finite, perhaps short, computational grid on which the wave packet has been defined. Define the disturbance in the new frame by P'', where

$$P(x,z,t) = P''(x'',z'',t''). \tag{10}$$

Proceeding as before we obtain

$$P_{xx} = P''_{x''x''} \tag{11}$$

$$P_{zz} = P''_{z''z''} - 2c^{-1}P''_{t''z''} + c^{-2}P''_{t''t''} \tag{12}$$

$$P_{tt} = P''_{t''t''}. \tag{13}$$

Inserting these into the wave equation (1) we obtain

$$P''_{t''z''} = (c/2)(P''_{x''x''} + P''_{z''z''}). \tag{14}$$

To see that the last term of (14) is small of higher order for waves travelling at small angles from the z-axis, recall that the solution to the wave equation for waves in the $+z$ direction is an arbitrary function $f(t - z/c) = f''(t'')$. Thus $\partial f''/\partial z''$ vanishes for a wave along the z'' axis. Neglecting $P''_{z''z''}$ we find that (14) reduces to

$$P''_{t''z''} = (c/2)P''_{x''x''} \tag{15}$$

which is the same as equation (8).

3. The difference approximation and a method of solution

Omitting primes from (8) or (15) we have

$$P_{tz} = (c/2)P_{xx}. \tag{16}$$

From a mathematical point of view this equation is completely symmetric with regard to t and z. For the sake of definiteness we will take the point of view of (15), namely, that P was defined at z_0 for all x and t and we are intending to use (16) to extrapolate the waveforms in the z-direction. Obviously we could also take the alternate point of view, that of equation (8). To express (16) in terms of sample time, as we must do for computation, the notion of a Z-transform is essential. A valuable introductory reference to Z-transforms is Treitel & Robinson (1964). First we put (16) into the frequency domain by replacing the time derivative by $-i\omega$.

$$-i\omega P_z = (c/2)P_{xx}. \tag{17}$$

Next we re-express the angular frequency variable ω in terms of the Z-transform variable

$$Z = \exp(i\omega\Delta t).$$

Taking the logarithm we have

$$i\omega\Delta t = \ln Z.$$

Using a well-known expansion for the logarithm we have

$$i\omega\Delta t = 2\left[\frac{Z-1}{Z+1} + \frac{1}{3}\left(\frac{Z-1}{Z+1}\right)^3 + \frac{1}{5}\left(\frac{Z-1}{Z+1}\right)^5 \dots\right].$$

By retaining only the first term we restrict the validity of our results to waveforms sampled moderately densely (say eight points/wavelength; see Fig. 3). Thus we take

$$-i\omega\Delta t = 2(1-Z)/(1+Z). \tag{18}$$

If P is taken to be a function sampled in the time domain then its Z-transform (which on the unit circle is its Fourier transform) takes the form

$$P = \dots p_{-1}Z^{-1} + p_0 + p_1 Z + p_2 Z^2 + \dots. \tag{19}$$

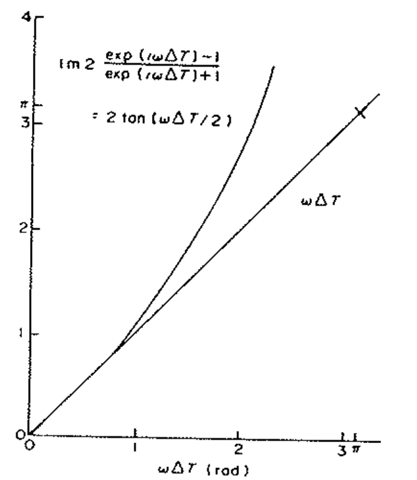

FIG. 3. This shows how the Z-transform approximation forces the data sampling rate to be at least four times the Nyquist sampling rate.

Without loss of generality for the application in mind we may ask P to vanish before $t = 0$, which means that (19) specializes to

$$P = p_0 + p_1 Z + p_2 Z^2 + \ldots = \sum_j p_j Z^j. \tag{20}$$

Putting (18) and (20) into (17) we have

$$\frac{2}{\Delta t} \frac{1-Z}{1+Z} \frac{\partial}{\partial z} P = \frac{c}{2} \frac{\partial^2}{\partial x^2} P. \tag{21}$$

As with all Z-transform equations one has an equation in the frequency domain if one regards Z as taking on all numerical values on the unit circle, and one has an equation at each point in the time domain if one identifies coefficients of various powers of Z. Next let us make the z co-ordinate discrete in equation (21); this follows the approach of earlier papers. Let $P(z)$ be denoted by $P^{n\Delta z}$ or more simply by P^n. Utilizing central differences (21) becomes

$$(1-Z)(P^{n+1} - P^n) = \frac{c\Delta t \Delta z}{4} (1+Z) \frac{\partial^2}{\partial x^2} \frac{P^{n+1} + P^n}{2}. \tag{22}$$

The reader will observe that to avoid defining $P^{n+1/2}$ we have used the average $(P^{n+1} + P^n)/2$. It is shown by Claerbout (1970a) that the more general form $\theta P^{n+1} + (1-\theta) P^n$ (where θ is the implicit/explicit parameter whose value lies between one-half and one) offers some advantage in attenuation of off-axis waves. This is an important parameter is practice but we set $\theta = \frac{1}{2}$ for simplicity since a satisfactory discussion of its use already was given by Claerbout (1970a).

Finally it remains to make (22) discrete with respect to the x-co-ordinate. Let p_j^n for fixed n (depth) and j (time) be called a vector because it contains an unwritten subscript which refers to variation in the x-direction. Then $\partial^2/\partial x^2$ is like a tri-diagonal matrix with the second-difference operator $(1, -2, 1)/\Delta x^2$ on the main diagonal. Denoting by T a tri-diagonal matrix with $(-1, 2, -1)$ on the main diagonal, (22) may be written

$$(1-Z)(P^{n+1} - P^n) = -\frac{c\Delta t \Delta z}{8\Delta x^2} (1+Z)\, T(P^{n+1} + P^n) \tag{23}$$

which we abbreviate by

$$(1-Z)(P^{n+1} - P^n) = -a(1+Z)\, T(P^{n+1} + P^n). \tag{24}$$

We bring terms depending on the disturbance at $(n+1)\,\Delta z$ to the left and the others to the right to get

$$[(1+aT) - Z(1-aT)]\, P^{n+1} = [(1-aT) - Z(1+aT)]\, P^n. \tag{25}$$

Recognizing that P^n and P^{n+1} represent Z-transform polynomials (20) of the disturbance at z and $z+\Delta z$, we may identify in (25) the coefficient of Z^{j+1}. For any $j > 0$ the coefficient is

$$(1+aT)\, p_{j+1}^{n+1} - (1-aT)\, p_j^{n+1} = (1-aT)\, p_{j+1}^n - (1+aT)\, p_j^n \tag{26}$$

which we may rearrange to

$$(1+aT)\, p_{j+1}^{n+1} = (1-aT)\, p_j^{n+1} + (1-aT)\, p_{j+1}^n - (1+aT)\, p_j^n. \tag{27}$$

Suppose for some particular n and j that everything on the right-hand side is known. Computationally the right side then represents a known vector of NX components where NX refers to the number of points which have been sampled on the x-axis. The factor $(1+aT)$ represents a tri-diagonal matrix of size NX. Thus we have a very

sparse set of simultaneous equations which may be solved (by the method of Richtmyer & Morton, 1967, 198–201, for example) for the vector p_{j+1}^{n+1}. Of course boundary conditions are required at the ends of the vector. The authors have thus far used zero-slope conditions at the extremes on the x-axis. This is satisfactory when either the function or its x-derivative is sufficiently small at the boundaries.

It is clear that a satisfactory initial condition for the recursion (27) is knowledge of both p_j^0 for all j and p_0^n for all n, both of course for all values of the x-co-ordinate $(k\Delta x)$. Let us consider an example of a point source at $t = 0$ and $z = -10\Delta z$. Clearly $p_0^n = 0$ for all $n \geqslant 0$. Also p_j^0 vanishes for enough values of j for the wave to have time to get to $z = 0$. After that p_j^0 is an arbitrary source waveform. By elementary geometry the x-dependence of p_j^0 must be worked out to conform to the well-known hyperbola in the $x-t$ plane.

4. Stability

In this section we will show that stability is assured for all values of Δx, Δz, and Δt. Stability is lost if the calculation is set up in an unnatural direction. For example, we know that waveforms move in the $+z'$ direction in the x'-z' plane. In other words information at $z' - \Delta z'$ will later be at z'. Thus it is reasonable and stable to calculate $P'(z')$ from present and past values of $P'(z' - \Delta z')$ but it is unreasonable and unstable to try to calculate $P'(z')$ from the present and past values of $P'(z' + \Delta z')$. To get $P'(z')$ from $P'(z' + \Delta z')$ it would be necessary to use present and future values.

First of all the stability of the recursion on x is assured because a is positive and the matrix $(1+aT)$ has $1+2a$ on the main diagonal and $-a$ off the main diagonal, so the matrix is diagonally dominant.

Next let us consider the recursion on time. Eigenvalues for the T matrix may be shown to lie between zero and $+4$. Thus we may consider T in (25) to be replaced by an arbitrary number between zero and $+4$. To determine P^{n+1} from P^n it is necessary to divide (25) by the left-side-polynomial $[(1+aT) - Z(1-aT)]$. This polynomial will be minimum-phase (Treitel & Robinson 1964) because a is positive, T is positive, and the coefficient of Z^0 always dominates the coefficient of Z^1. Notice that the polynomial on the right-hand side of (25) will be definitely not be minimum phase, so that one cannot determine P^n from P^{n+1}.

Finally we show stability with regard to stepping in the z-direction. For this it is satisfactory to show that the transfer function of (25), namely

$$\frac{(1-aT) - Z(1+aT)}{(1+aT) - Z(1-aT)}$$

has unit magnitude for all frequencies (Z on the unit circle) and all horizontal wavelengths $(0 \leqslant T \leqslant 4)$. The transfer function is of the form $(b-cZ)/(c-bZ)$. Since $Z = \exp(i\omega\Delta t)$ we have $Z^{-1} = \exp(-i\omega\Delta t)$. Therefore multiplying the transfer function by its conjugate we have

$$\frac{b-cZ}{c-bZ} \cdot \frac{b-c/Z}{c-b/Z} = \frac{b^2+c^2-bc(Z+1/Z)}{b^2+c^2-bc(Z+1/Z)} = 1.$$

From the point of view of stability with respect to extrapolation in the z direction it is irrelevant whether we go from P^n to P^{n+1} or the reverse. It is the recurrence on time which must go in one particular direction. Finally we remind the reader that z and t, n and j, and ω and k_z (vertical wave number) may be interchanged if they are interchanged *everywhere* throughout the discussion subsequent to equation (16).

5. Example—disturbed plane wave

While most theoretical solutions in wave propagation deal with highly symmetric waves (plane, cylindrical, etc.), nature is seldom so regular. The example shown in Fig. 4 represents a disturbed plane wave which might have been produced as shown in Fig. 5. At the bottom of Fig. 5 a plane atmospheric pressure wave is incident upon a series of circulating cells which tend to advance the wave in some regions while retarding it in other regions, thus producing the waveform shown at the top of Fig. 5. A somewhat similar situation is the phase grating in optics (Goodman 1968, p. 69), where the monochromatic solution is usually obtained at infinity.

The seven frames in Fig. 4 illustrate the subsequent development of the disturbed wave shown in frame No. 1. Note the frames may be thought of interchangeably in either steps of $\Delta t'$ or $\Delta z''$, as has been discussed previously. The most obvious development is that the energy spreads out as one moves down the figure. The single pulse of the top frame has become an extended oscillatory arrival by the last frame. As time goes on, less and less energy is in the first pulse and more and more is in the oscillatory tail. Note: in the figure the gain is adjusted with each frame to give better contrast. Also, as might be expected, the wave onset time, which is a dramatic function of x in the first frame, is nearly independent of x by the last frame. A surprising feature is that although energy moves back from the first arrival (see Figs 1 and 2) a point of constant phase in the wave tail moves forward toward the first arrival (a point of constant phase is marked by an X on the right side of each frame). Another clear feature of the wave tails is that dip (arrival time dependence on x) increases going down a single frame. The phase shift of the two-dimensional focus, which causes doublets to form, is clearly visible at the point in frame No. 2 marked with an A.

In order to represent a disturbance of infinite extent in x on a finite computer grid, we initialized the problem with a periodic disturbance having zero slope at the side boundaries. Zero-slope boundary conditions are then equivalent to infinite periodic extension in x. A value $a = \frac{1}{4}$ was chosen to given an appropriate variation in progressive frames, with each frame in Fig. 4 representing 5 computational iterations. The solution may be rescaled in several ways due to the interdependence in the constant a (see equation (24)) of $c\Delta t$, Δx and Δz. The calculation in Fig. 4 required about 0·5 min on the IBM 360/67.

It might be valuable to consider various data enhancement processes in the light of Fig. 4. In the process called 'beam-steering', observations as in Fig. 4 would be

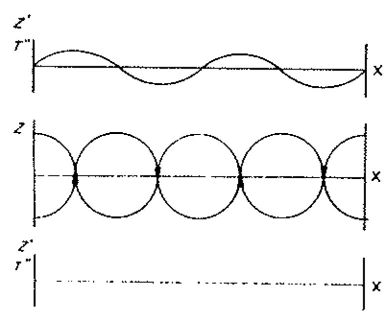

Fig. 5. One means of producing a disturbed plane wave. Incident plane wave at bottom is altered by a material inhomogeneity (centre), resulting in the disturbed wave front at top.

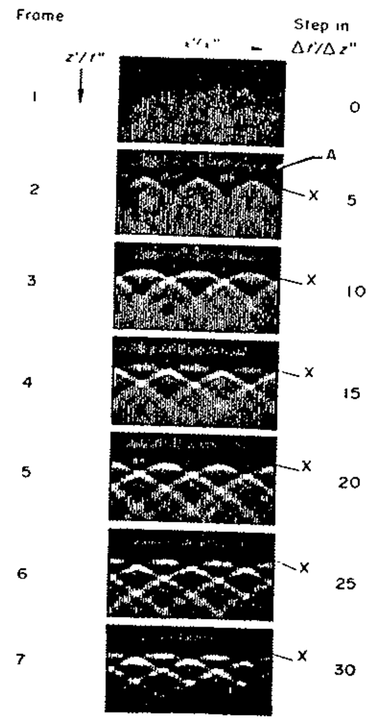

Fig. 4. Disturbed plane wave propagating through a homogeneous space in a moving co-ordinate system. Features of note are: (1) energy moves backwards and toward the sides; (2) the wave onset is a dramatic function of x near the beginning, but by the last frame is nearly independent of x; (3) phase of the wave tail (marked by x in each frame) moves forward; (4) the dip of the wave tail increases in a given frame as one scans down the frame; (5) in frame No. 2, the letter A indicates a two-dimensional focus.

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summed over the x-co-ordinate in an effort to enhance signal and reject noise. Clearly beam-steering will enhance the first arrival while rejecting random noise. What it will also do is to tend to cancel the signal energy which resides in the oscillatory wave tails. If one is really interested in enhancing signal-to-noise ratio it would hardly seem desirable to use a processing scheme which cancels signal energy. As z' or t' is increased the situation becomes increasingly severe since signal energy moves from the initial pulse toward the oscillatory wave tails. The central practical conclusion of this section is that what has often been regarded as 'signal-generated-noise' may turn out to be signal in a potentially valuable form. We can, indeed, expect dramatic results if we are able to learn how to design data enhancement techniques on entire waveforms rather than on the initial pulse alone.

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DOWNWARD CONTINUATION OF MOVEOUT-CORRECTED SEISMOGRAMS†

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Earlier work developed a method of migration of seismic data based on numerical solutions of partial differential equations. The method was designed for the geometry of a single source with a line of surface receivers. Here the method is extended to the geometry of stacked sections, or what is nearly the same thing, to the geometry where a source and receiver move together along the surface as in marine profiling. The basic idea simply stated is that the best receiver line for any reflector is just at (or above) the reflector. Data received at a surface line of receivers may be extrapolated by computer to data at a hypothetical receiver line at any depth. By considering migration before stacking over offset, it is found that certain ambiguities in velocity analysis may be avoided.

THE SIMPLEST CASE

For the sake of clarity we begin with the simplest case for which migration is a useful concept. Then realistic complications can be included in order of their practical importance. First, consider a two-dimensional model of the earth, y horizontal, z downward, in which the seismic velocity c is constant but the reflectors have arbitrary dips and curvatures. Here we neglect shear waves, although they can be treated by the method of Landers and Claerbout (1972). We also neglect multiples and energy spreading into the third dimension. Multiples will be treated in a later paper. Let there be sources and receivers uniformly spaced over the y axis at intervals of Δy. We can suppose that all of our shots are set off in unison. (Even if they are not we may synthesize it in a computer by adding seismograms together.) At some depth which is very great compared to Δy the semicircular wavefronts will combine together in the fashion of Huygens secondary sources to make a downgoing wave which is essentially a plane wave. In other words, at sufficient depth, point sources along the surface are indistinguishable from a surface line source. It turns out that in practice there are usually not enough shots and deep enough reflectors for the plane-wave approximation to be very good. Nevertheless, this is a useful starting point and we will return later to consider the fact that the downgoing wave is not a plane wave.

Considering the downgoing wave to be merely an impulsive plane wave simplifies the task of migration because we then may turn our attention entirely to the upgoing wave; since, according to the basic principle (Claerbout, 1971b)

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"reflectors exist at points in the earth where the first arrival of the downgoing wave is time coincident with an upgoing wave."

A useful departure from our earlier work is that we are now migrating data from many shotpoints at a time, whereas previously each shotpoint was migrated separately before summation. The major effort in migration is to take the observed upgoing wave at the surface and project it back downward. A frequency-domain technique for this downward projection is given in Claerbout (1970 and 1971b). Here we will give a time-domain method because it leads to our central topic of downward continuation of moveout-corrected seismograms.

We are basically interested in projecting upgoing waves back into the earth. As pointed out in earlier papers, great economy and stability can be achieved by specializing the wave equation, which is second order and has both upgoing and downgoing solutions, to a first-order equation with only upgoing solutions. The first step is to re-express the scalar wave equation in a coordinate frame which translates upward with the speed c. In such a moving coordinate frame, the upcoming waves will be Doppler shifted to lower frequencies and the downgoing waves will be Doppler shifted to higher frequencies. Then a low-pass filtering type of operation can separate up from downgoing waves.

We have the scalar wave equation

$$0 = \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2}, \tag{1}$$

which we abbreviate as

$$0 = P_{yy} + P_{zz} - c^{-2}P_{tt}. \tag{2}$$

We have the transformation to a coordinate frame translating upward with velocity c

$$y' = y, \tag{3a}$$

$$z' = z + ct, \quad \text{and} \tag{3b}$$

$$t' = t, \tag{3c}$$

and we have the statement that the new coordinate frame contains the same wave disturbance as the old frame.

$$P(y, z, t) = P'(y', z', t'). \tag{4}$$

To express the wave equation (2) in terms of the translating coordinate system, we may use the chain rule for partial differentiation. Obviously, $P_y = P'_{y'}$ and

$$P_{yy} = P'_{y'y'}. \tag{5}$$

Also

$$P_z = P'_{y'}y'_z + P'_{z'}z'_z + P'_{t'}t'_z = P'_{z'},$$

so

$$P_{zz} = P'_{z'z'}. \tag{6}$$

The nontrivial differentiation is

$$P_t = P'_{y'}y'_t + P'_{z'}z'_t + P'_{t'}t'_t = cP'_{z'} + P'_{t'},$$

which on a second differentiation leads to

$$\begin{aligned} P_{tt} &= c(cP'_{z'z'} + P'_{t'z'}) + (cP'_{z't'} + P'_{t't'}) \\ &= c^2P'_{z'z'} + 2cP'_{t'z'} + P'_{t't'}. \end{aligned} \tag{7}$$

Now we insert (5), (6), and (7) into the scalar wave equation (2) and obtain the scalar wave equation in a translating frame,

$$0 = P'_{y'y'} - \frac{2}{c}P'_{t'z'} - c^{-2}P'_{t't'}. \tag{8}$$

The rightmost term $P'_{t't'}$ is proportional to the square of the Doppler-shifted frequency of a wave. Thus, dropping this term may be expected to have little effect on upgoing waves but a drastic effect on downgoing waves. In fact, dropping the last term of (8) has the desired effect of eliminating the downgoing waves altogether as can be seen by comparing these results to earlier work (Claerbout 1970, 1971a, b). Simply dropping the term does have the undesired effect of limiting velocity accuracy to about a percent at 15 degrees. Economical procedures for obtaining better than a percent accuracy at 45 degrees also may be found in the earlier work.

Dropping the last term from (8), we have

$$P'_{t'z'} = \frac{c}{2}P'_{y'y'}. \tag{9}$$

A computer algorithm for solving (9) is described in Claerbout and Johnson (1972) along with a more detailed discussion including accuracy, stability, and an air-wave example.

Knowing the form of the upcoming wave at the surface of the earth, we can use equation (9) to project the upgoing wave backwards in time and, thus, find the upgoing wave at greater and greater depths. Using the basic principle "reflectors exist at points in the earth where the first arrival of the downgoing wave is time coincident with an upgoing wave," and considering the downgoing wave to be a delta function at time $t = z/c$, we get for the earth structure $S(y, z)$

$$S(y, z) = \int P(y, z, t)\delta(t - z/c)dt, \tag{10}$$

where $P(y, z, t)$ is found by solving (9) for $P'(y', z', t')$ and then transforming $P'(y', z', t')$ to $P(y, z, t)$ with the inverse to equation (3).

MOVEOUT-CORRECTED SEISMOGRAMS

In the previous section we considered the data which would be recorded if all surface shots were set off at the same time. If the shots are not set off at the same time but data is recorded from each shot separately, simultaneous shooting can by synthesized in a computer by stacking the data. For each receiver all seismograms of the different shots are aligned by shot time and added together. In practice, moveout correction would be applied before stacking. This correction is intended to remove source-receiver geometrical effects. Since it was ignored in the previous section, the results were limited to very great depths where the correction is small.

Figure 1 defines a moveout corrected profile of seismic data. Great improvement in the partial differential-equation migration method results from the idea that an NMO profile and often a zero-offset section can be governed by a differential equation. Figure 1 illustrates how an NMO profile M may be constructed from an upcoming wave U by transcribing data values from the (y, t) plane to the (x, d) plane. This operation is actually a coordinate transformation of the data. Since the moveout-corrected profile is just the upcoming wave with coordinate axes deformed, it is not surprising that moveout-corrected profiles can be governed by an equation derived through a coordinate stretching transformation of the wave equation. The idea that there should be any advantage to using differential equations on synthetic things like moveout-corrected profiles (as compared to natural things like waves) arose out of the following observations: It is comparatively inefficient to let a wave packet propagate across a grid in a computer. It is much more efficient to describe a wave in a coordinate frame which moves along with the wave, as was done for the surface line source case in the beginning of this paper. In such a frame, things happen slowly and larger time increments may be used. There is a similar situation in a nearly layered medium where seismic arrivals (upcoming waves) fit nearly hyperbolic traveltime curves, and the object of a migration program is to deform the hyperboloids into lines which represent the layer-

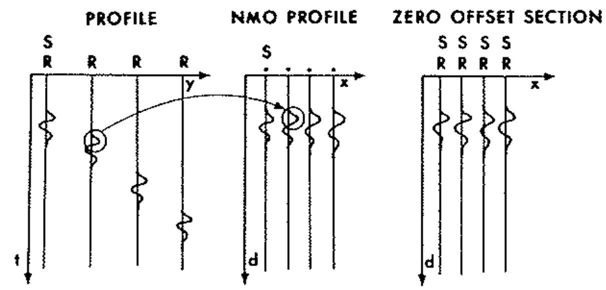

FIG. 1. Definition of profile and section. The left frame is indicative of observed upcoming waves recorded with one shot and a surface receiver line. This is called a *profile*. Data points are moved from this profile $(y, t$ plane$)$ to the central frame $(x, d$ plane$)$ under control of transformation equations like (13), with $z' = 0$. The central frame will be called an *NMO profile*. The third frame depicts upcoming waves recorded with a different shot receiver geometry. Each recording is made with the shot and receiver at the same location. This will be called a (zero-offset) *section*.
Although the methods of this paper assume the geometry of the *NMO profile*, they will often be applicable to data recorded as *sections*.

ing. A migration partial differential equation does a lot of work just moving energy around on a grid from one more or less predictable place to another. Considerable effort can be saved by doing moveout correction (including application of a time-variable velocity if necessary) to get energy in approximately the right place before requiring a differential equation to migrate the data to its final proper position. If reflecting layers are perfectly flat and level, migration makes no change to the moveout-corrected data. The greater the structural dips and curvatures, the greater will be the task for the migrating differential equation. Grid spacing can be chosen according to the maximum anticipated dip.

Although efficiency was the motivation in the search for a differential equation to control migration from moveout-corrected data, there are two concomitant benefits which are far more important than efficiency. First, the data from spatially separated shotpoints may be stacked before migration, thereby enhancing signal-to-noise ratio. Second, we often may dispense altogether with the line of surface receivers and migrate data recorded in the geometry, where shotpoint and receiver point move together across the earth's surface as in the simplest type of marine profiling.

By "downward continuation of moveout-corrected seismograms" we mean that beginning with moveout-corrected data observed at the surface, we will synthesize moveout-corrected seismograms corresponding to hypothetical receivers at successively increasing depths.

One reason for wanting the moveout-corrected data for buried receivers is that it is related, through geometry, to the upcoming wave which is needed to make a migrated profile. Another reason, which is really the same, relates to the nature of seismic diffraction. Figure 2 (after Hilterman, 1970) illustrates that a data section can be expected to resemble a cross-section through the reflector if the radius of curvature of the reflector is much greater than the distance from the reflector to the receiver. Otherwise, one has a buried focus or diffraction. (Technically, a diffraction is a limiting case where some radius of curvature of a structure goes to zero.) In other words, moveout-corrected data gives a better representation of a structure if the receivers are near the structure, than if they are far away. In fact, when the receivers are at the depth of the structure the buried focus problem disappears altogether. Diffractions from point scatterers also collapse to points when the receivers are at the same depth as the scatter. The method of migration proposed here is that as data are projected to successively greater depths, that part of the data corresponding to the receiver depth is set aside as belonging to the migrated data at that depth. Thus, various depths on the depth section are developed in succession as the moveout-corrected data are projected downward.

To be precise about the meaning of moveout-corrected data for buried receivers, we refer to Figure 3. Since the moveout-corrected profile M is created by a coordinate stretching of the upcoming wave U, we have

$$M(x, d, z') = U(y, t, z). \tag{11}$$

Observe the conceptual similarity of the relationship between $U(y, t, z)$ and $M(x, d, z')$ to the relationship between $P(y, z, t)$ and $P'(y', z', t')$. The fact that P and P' are the same thing expressed in different coordinates is analogous to the fact that U and M are the same thing expressed in different coordinates. At the surface $z=0$ we record the upcoming wave $U(y, t, 0)$, and using a presumed velocity (which need not be precisely correct), we transform axes to the NMO profile $M(x, d, 0)$. The downward (z) continuation of receivers of $U(y, t, z)$ with the wave equation will be equivalent to downward (z') continuation of $M(x, d, z')$ with an equation we are about to derive. Although these two different downward continuations would be expected to give the same results, i.e., one could transform from U to M or M to U at any depth, there are several reasons to prefer downward continuation with M: 1) profiles from various shotpoints may be summed before downward continuation, 2) a coarser grid mesh may be used, and 3) sections may be downward continued.

To obtain the differential equation for U, first we must define the coordinate transformation from (y, t, z) to (x, d, z') and then use the chain rule to compute the required partial derivatives. From Figure 3 by means of elementary geometry, one may deduce the transformation

$$y(x, d, z') = x\cdot(2 - z'/d), \tag{12a}$$

$$t(x, d, z') = (2d - z')(1 + x^2/d^2)^{1/2}/c, \tag{12b}$$

and

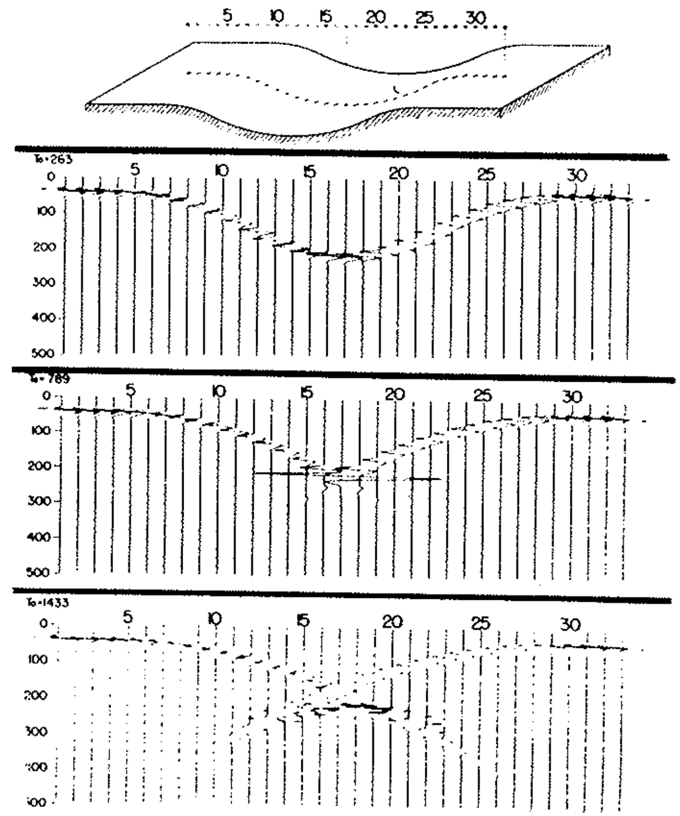

FIG. 2. (After Hilterman, 1970) Time sections recorded at various heights above a model. The top section (T_0 = 263) has shotpoint below the curvature axis, center section at the curvature axis, and bottom section above the curvature axis.

$$z(x, d, z') = z', \tag{12c}$$

and, by means of tedious algebra, the inverse transformation is found to be

$$d(y, t, z) = (z + (c^2t^2 - y^2)^{1/2})/2, \tag{13a}$$

$$x(y, t, z) = (y/2)(1 + z/(c^2t^2 - y^2)^{1/2}), \tag{13b}$$

and

$$z'(y, t, z) = z. \tag{13c}$$

In constant velocity material one could find an equation for M which is valid for all offsets x. The algebra would be overwhelming, so we make the simplifying practical assumption that $x/d \ll 1$. The authors were surprised to discover that even if offset terms like x/d or y/t are completely neglected one still obtains a result which is a big improvement over equation (9) of the introductory section. The reason is that even as offset goes to zero, the ratio of x to y remains important. Although (12) and (13) require a somewhat careful and detailed deduction, the zero offset relations are much easier and will be shown in detail. By

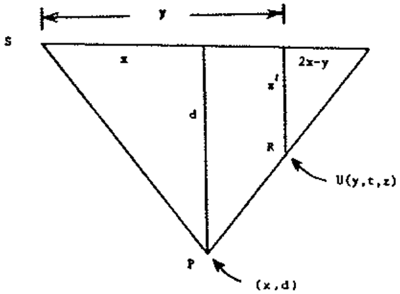

FIG. 3. Moveout correction geometry for buried receivers. There is a surface shot S and buried receiver R. The wave is assumed to reflect at the point P. The receiver R at (y, z) measures the upcoming wave $U(y, t, z)$. From the upcoming wave a moveout corrected profile $M(x, d, z')$ is constructed by axis stretching according to the geometry of the raypath SPR.

similar right triangles, the ratio of x to d is the same as the ratio $2x-y$ to z'. Thus $x/d=(2x-y)/z'$ or

$$y(x, d, z') = x(2 - z'/d). \quad (14a)$$

The traveltime along SP will be d/c and along PR it will be $(d-z')/c$. Thus,

$$t(x, d, z') = (2d - z')/c. \quad (14b)$$

The zero offset limit of (12) is (14). Now for the inverse relations solve (14b) for d and use $z'(y, t, z)=z$,

$$d(y, t, z) = (ct + z)/2. \quad (15a)$$

Solve (14a) for x eliminating d with (15a) and using $z'=z$,

$$x = y/(2 - z/d)$$

and

$$x(y, t, z) = y/[2 - 2z/(ct + z)]. \quad (15b)$$

From the coordinate transformation (14) and its inverse (15), it will be an easy matter now to compute the partial derivatives required for the transformation of the upgoing-wave equation. One derivative of particular interest, x_y, is computed from (15b) to be

$$x_y = 1/(2 - 2z/(ct + z)),$$

which in terms of the other variables is

$$x_y = 1/(2 - z'/d) = d/(2d - z'). \quad (16)$$

Computing all the partial derivatives at the zero offset limit and arranging into a matrix, we have

$$\begin{bmatrix} x_y & x_t & x_z \\ d_y & d_t & d_z \\ z'_y & z'_t & z'_z \end{bmatrix} = \begin{bmatrix} d/(2d - z') & 0 & 0 \\ 0 & c/2 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}. \quad (17)$$

Now there are two possible ways to proceed. The simplest way is to use the chain rule and insert partial derivatives into the wave equation. Another way is to insert the partial derivatives into the upcoming-wave equation. Let us insert the partial derivatives into the wave equation (2).

$$U_{yy} + U_{zz} - c(y, z)^{-2}U_{tt} = 0. \quad (18)$$

We must be aware that (18) also has downgoing solutions which we will later eliminate by dropping a second order z' derivative. Recalling (11) that $U(y, t, z)=M(x, d, z')$, we compute the

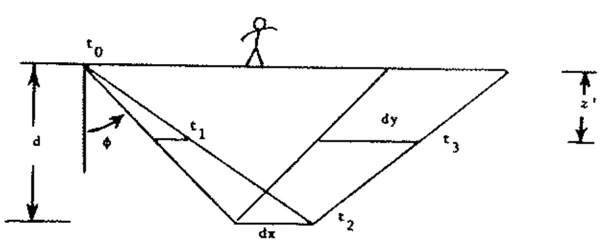

FIG. 4. Geometry for calculation of $(\partial x/\partial y)_t = 1/(\partial y/\partial x)_d$.

necessary terms for insertion into (18). Before insertion we simplify with the zeros in (17) and with the high-frequency approximation (gradients of waves are taken to be much greater than gradients of the coordinate transformation coefficients).

$$U_t = M_x x_t + M_d d_t = M_d d_t, \quad (19)$$

$$U_{tt} = M_{dd} d_t^2, \quad (20)$$

$$U_y = M_x x_y + M_d d_y = M_x x_y, \quad (21)$$

$$U_{yy} = M_{xx} x_y^2, \quad (22)$$

$$U_z = M_{z'} + M_x x_z + M_d d_z = M_{z'} + M_d d_z, \quad (23)$$

$$U_{zz} = M_{z'z'} + 2d_z M_{dz'} + M_{dd} d_z^2, \quad (24)$$

and

$$c(y, z) = c[y(x, d, z'), z(x, d, z')]. \quad (25)$$

Inserting (20), (22), (24), and (25) into (18) we obtain for the downward projection of the moveout-corrected data

$$M_{xx}x_y^2 + M_{z'z'} + 2M_{dz'}d_z + M_{dd}(d_z^2 - c^{-2}d_t^2) = 0, \quad (26)$$

which, utilizing (17) and the assumption that the true (wave-equation) velocity equals the constant stacking velocity (in d_t), simplifies to

$$M_{dz'} = -\left(\frac{d}{2d - z'}\right)^2 M_{xx} - M_{z'z'}. \quad (27)$$

The last term arises mainly because the wave equation we started from also has downgoing waves. The omission of this kind of term is discussed in the first section and in earlier papers. There is a formal similarity between (9) and (27), so the same computer algorithm may be used.

VARIABLE VELOCITY

The result of the last section, equation (27), is restricted to material of constant velocity. It will be useful in practice to generalize the result to space-variable velocity. First note that *the velocity $\hat{c}$ in the wave equation (18) need not be the same as the velocity, say $\bar{c}$, in the moveout-correction equations* (11) to (17). It can be shown that the wave equation is still valid at high frequencies ($\omega/c \gg |\nabla c|/c$) when the velocity $\hat{c}$ is space variable, although the moveout-correction equations must be modified if the velocity $\bar{c}$ is to be space variable. Thus, (26) is a valid equation for downward continuation in variable-velocity material for data stacked at a constant velocity c. The principal change to (27) resulting from space-variable velocity is that the M_{dd} term does not drop out. The difficulty which arises if $c \neq \hat{c}$ is not in the differential equation (26) but in the fact that there may be destructive interference when stacking with the wrong velocity. Therefore we will recompute the partial-derivative matrix (17) for arbitrary depth dependence of stacking (moveout-correction) velocity. This will require some care. The matrix of (17) is really the Jacobian matrix of the transformation from (y, t, z) variables to (x, d, z') variables, that is:

$$\begin{bmatrix} dx \\ dd \\ dz' \end{bmatrix} = \begin{bmatrix} x_y & x_t & x_z \\ d_y & d_t & d_z \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dy \\ dt \\ dz \end{bmatrix}. \quad (28)$$

The inverse transformation may be defined as

$$\begin{bmatrix} dy \\ dt \\ dz \end{bmatrix} = \begin{bmatrix} y_x & y_d & y_{z'} \\ t_x & t_d & t_{z'} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dx \\ dd \\ dz' \end{bmatrix}. \quad (29)$$

The statement that (29) is indeed inverse to (28) is

$$\begin{bmatrix} x_y & x_t & x_z \\ d_y & d_t & d_z \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_x & y_d & y_{z'} \\ t_x & t_d & t_{z'} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (30)$$

Near the zero offset limit x and y tend to zero and are, therefore, independent of t, d, and z, so that $x_t = x_z = y_d = y_{z'} = 0$. Also, a small change in x or y has a second-order effect on traveltime so $t_x = d_y = 0$. Thus, at small enough offsets (30) reduces to

$$\begin{bmatrix} x_y & 0 & 0 \\ 0 & d_t & d_z \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_x & 0 & 0 \\ 0 & t_d & t_{z'} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (31)$$

The interesting parts of (31) are

$$x_y = 1/y_x, \quad (32)$$

$$d_x d_d = 1, \quad (33)$$

and

$$d_x d_{z'} + d_z = 0. \quad (34)$$

To compute the partial derivative x_y, refer to Figure 4 which, for clarity, exaggerates the offset. For rays which are essentially vertical

$$dz = c\,dt. \quad (35)$$

We have Snell's law,

$$(\sin\phi)/c = \text{const} = p. \quad (36)$$

Tracing rays in time gives

$$x = \int_0^{t_2} c \sin\phi\, dt = p\int_0^{t_2} c^2 dt,$$

and using (35),

$$x = p\int_0^d c\,dz \quad (37)$$

and

$$y = p\int_0^{t_2} c^2 dt,$$

which, from Figure 4, may be written as

$$y = p\left(2\int_0^d c\,dz - \int_0^{z'} c\,dz\right). \quad (38)$$

It is convenient to use d and z' as superscripts to indicate integration in the same way we use them as subscripts to denote differentiation. Thus, we define

$$c^d = \int_0^d c(z)dz,\; c^{z'} = \int_0^{z'} c(z)dz,$$

and use (37) to eliminate p from (38);

$$y = x(2c^d - c^{z'})/c^d. \quad (39)$$

Differentiating with respect to x holding d and z' constant, gives

$$y_x = (2c^d - c^{z'})/c^d. \quad (40)$$

Thus, using (32),

$$x_y = c^d/(2c^d - c^{z'}). \quad (41)$$

In the constant velocity limit (41) obviously reduces to (10).

The partial derivatives of traveltime t with respect to receiver depth and with respect to recording position are found in a similar manner. The traveltime dt through a layer of thickness dz is given by

$$dt = dz/c(z)\cos\phi,$$

where ϕ is the angle from the ray to the vertical. Replacing $1/\cos\phi$ by $1+\sin^2\phi/2$, we have

$$dt = \left(\frac{1}{c(z)} + \frac{\sin^2\phi}{2c(z)}\right)dz.$$

Using (36) and (37) to second order in x, we get

$$dt = \left(\frac{1}{c(z)} + \frac{x^2 c(z)}{2(c^d)^2}\right)dz.$$

Referring to Figure 4 we note that the traveltime is twice the time from 0 to d, less the time from 0 to z'. Thus,

$$t = 2\int_0^d \left[\frac{1}{c(z)} + \frac{x^2 c(z)}{2(c^d)^2}\right]dz - \int_0^{z'}\left[\frac{1}{c(z)} + \frac{x^2 c(z)}{2(c^d)^2}\right]dz.$$

This equation may be used to find the partial derivatives of traveltime to first-order accuracy in x. However, in this section we are interested in finding the derivatives to zero order in x, so the terms proportional to x^2 may be omitted, giving

$$t = 2\int_0^d \frac{1}{c(z)}dz - \int_0^{z'} \frac{1}{c(z)}dz,$$

which we abbreviate as

$$t = 2(c^{-1})^d - (c^{-1})^{z'}. \quad (42)$$

Differentiating (42) with respect to d holding x and z' fixed, gives

$$t_d = +2/c(d), \quad (43)$$

where $c(d)$ is velocity $c(z)$ evaluated at $z = d$.

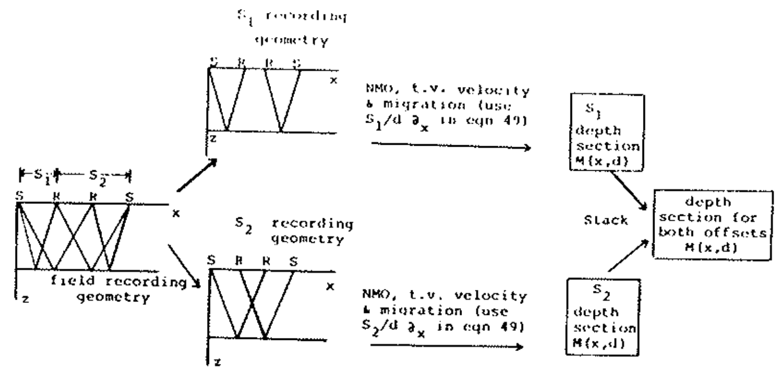

FIG. 5. Procedure for migrating data with several offsets. Field data are recorded using two receiver offsets, S_1 and S_2. The field data are separated into two sets: data recorded with offset S_1 and data recorded with offset S_2. Moveout corrections are done for both sets of data. Then the S_1 data are migrated with equation (49) with $(S_1/d)\partial_x$ substituted for $(x/d)\partial_x$, giving the S_1 depth section, $M(x,d)$. Next the S_2 data are migrated with equation (49) with $(S_2/d)\partial_x$ substituted for $(x/d)\partial_x$, giving the S_2 depth section, $M(x,d)$. Finally, the S_1 depth section and S_2 depth section are stacked to give the composite depth section.

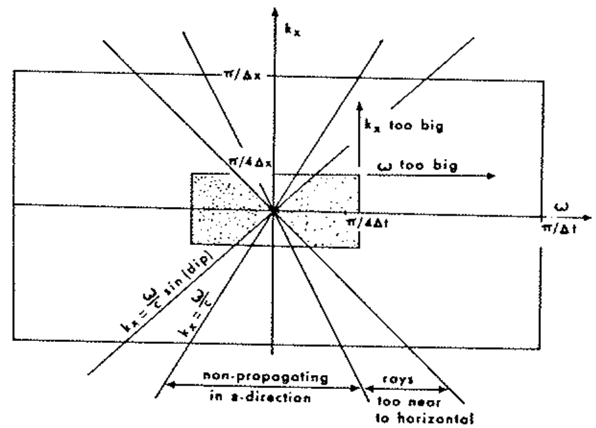

FIG. 6. The (ω, k) plane. Field data may be expected to have some energy everywhere in the (ω, k) plane. Only in the speckled region will our difference equations properly simulate the wave equation. Energy with $|k| > |\omega/c|$ does not represent free waves; it represents either surface waves or errors in data collection (often statics, random noise, or gain not smoothly variable from trace to trace). Such energy can mean nothing in a migration program, hence it should be rejected by filtering. This may be done by fan filtering (as in Treitel et al, 1967) or as was done here by means of numerical viscosity (Claerbout, 1970). Actually, for practical reasons one frequently may wish to reject rays outside a certain dip angle. This gives the larger fan filter reject region $|k| > |\omega/c \sin(\text{dip})|$. In fact our present implementation is inadequate for dips greater than about 45 degrees (Claerbout, 1971). Although information can be carried up to the folding frequency in both ω and k, in practice the use of operators of finite length narrows the useful bandwidth. As shown in Claerbout and Johnson (1972) the use of simple difference operators results in a practical bandwidth restriction to about a quarter of the folding frequency. This presents no problem in principle; data may be interpolated before processing, or more elaborate (i.e., longer) difference operators may be used. Finally, the figure was drawn with $\Delta x > c\Delta t$ because it represents the usual case in practice where extra points in time are more cheaply obtained than extra points in space.

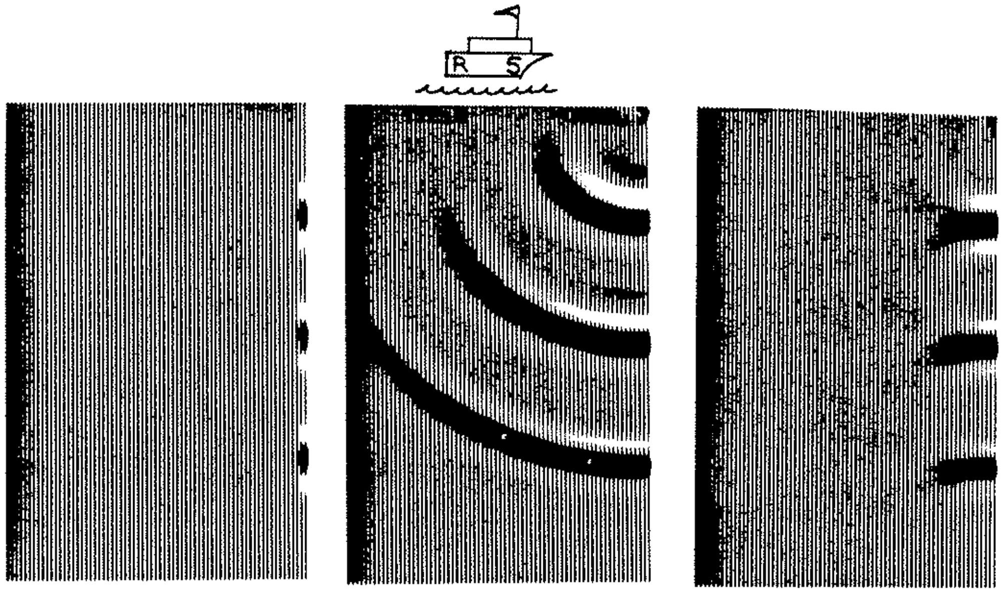

FIG. 7. The depth response to time-domain impulses and reconstruction of the impulses. The fact that the left frame is mostly blank depicts a situation in which no echo is received when a source and receiver move together in the horizontal direction until they reach the right-hand edge of the frame where the three blips indicate that there are three echoes at successively increasing times. With this as observed data, the logical conclusion is that the reflection structure of the earth is three concentric circles with centers on the right margin. The central frame shows the circles. (For economy the right edge of the frame is a plane of symmetry.) It will be noticed that the bottom of the circles is darker than the top. This is indicative of the 45-degree phase shift of bringing two-dimensional waves from a focus away from the focus. Waves with dips greater than about 45 degrees have been filtered away by numerical viscosity. The loss of this energy plus the loss of the energy of waves which propagate at complex angles results in a reconstruction (right-hand frame) in which the impulses are somewhat spread out in the horizontal direction.

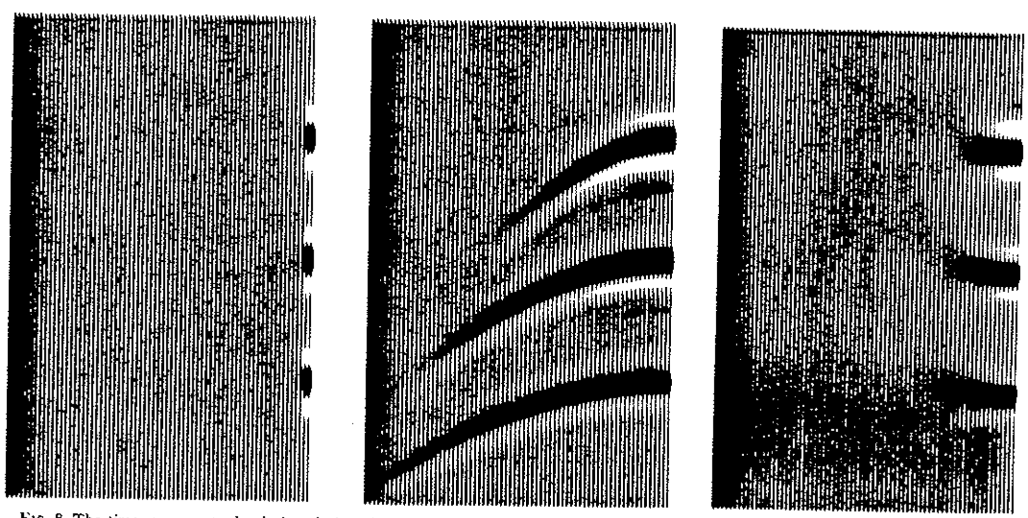

FIG. 8. The time response to depth-domain impulses and reconstruction of the impulses. The left frame depicts a model of the earth which consists of three point scatterers beneath one another along the right-hand edge. The second frame is the synthetic time data created from the model. Basically one observes the hyperbolic traveltime curves to the reflecting points. The third frame represents migration of the synthetic data back to the point scatterers. As in Figure 6 there is a reduced resolution because, in principle, horizontal resolution cannot be better than vertical resolution (which is controlled by the frequency content of the waves) and in practice we have included only rays up to angles of about 40 degrees.

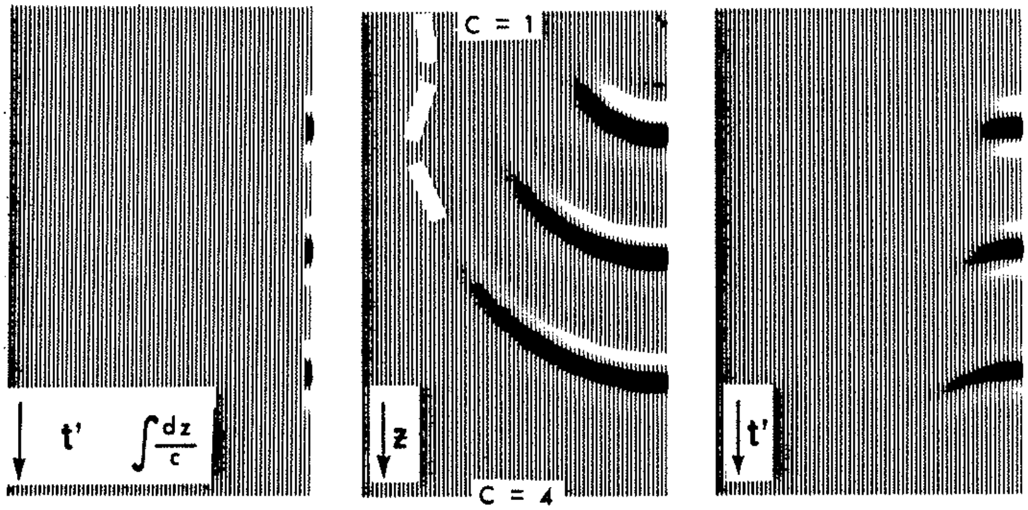

FIG. 9. Response to time-domain impulses in a velocity gradient. The gradient of inverse velocity is constant such that the velocity ranges over a factor of 4 from top to bottom of the frames. The first frame appears like the uniformly spaced (say 1, 2, 3) pulses of Figure 7. However, here the vertical coordinate is not time t but

$$t' = \int c^{-1} dz = \int (az + b) dz.$$

Thus the pulses are really spaced at times proportional to (.9, 1.6, and 2.0). The second frame represents the depth section. The circles of Figure 7 have changed to a narrower shape. If the rays were not cut off around 45 degrees, the shapes would resemble roughly that of a light bulb as indicated by the dashes. The third frame shows the reconstruction of the time data. It clearly deteriorates with depth. This is probably a result of the fact that a 40-degree beam halfwidth at the bottom reflector collapses to a 10-degree beam halfwidth at the surface because of ray curvature. There is also some erroneous bias in the reconstruction (the pulses "bend" a little). This is probably a result of inadequate dip filtering. At the time of writing we are uncertain how much the resolution loss in a velocity gradient results from our method and how much is fundamental.

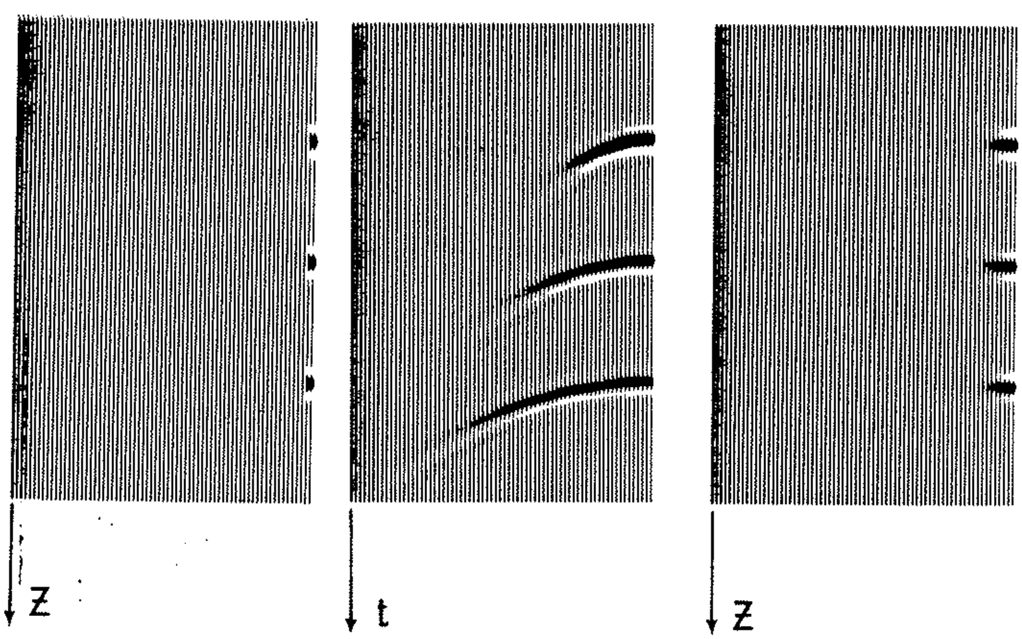

FIG. 10. This is like Figure 8 but the computer grid was not quite fine enough. Note "tails" on the ends of the hyperbolas. The resulting error is explained in the frequency domain in Figure 6. Note that the reconstruction is unaffected. Thus, the ability to reconstruct does not assure a high-quality migration.

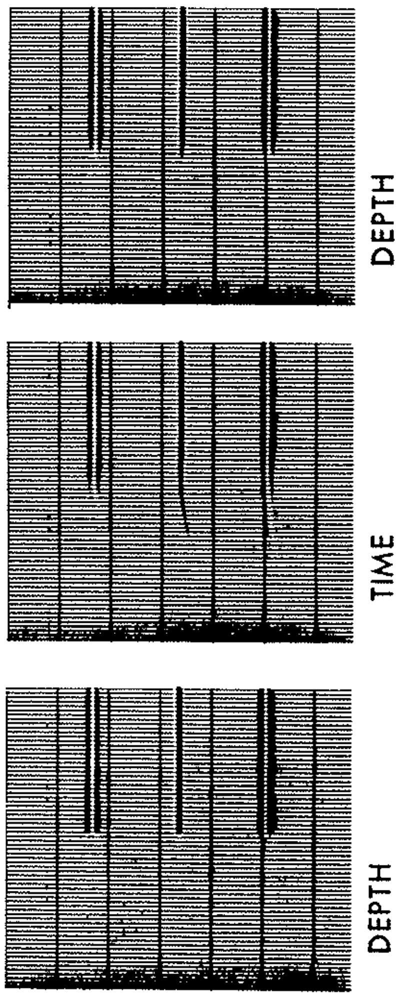

FIG. 11. Acoustic response to terminating interfaces and reconstruction. The left frame illustrates the (x, z) plane with three terminating interfaces. The central frame represents the synthetic "sparker-profile" data $[(x, t)$ plane]. The left branches of the hyperbolas have the polarity of the interface, but the right branches have the opposite polarity. This phenomenon was predicted by Trorey (1970). The rightmost frame is the attempted reconstruction of the model in the first frame.

Differentiating (42) with respect to z' holding x and d fixed, gives

$$t_{z'} = -1/c(z'). \tag{44}$$

Now we can proceed to find d_t and d_z. Substituting (43) into (33) we find

$$d_t = \frac{c(d)}{2}. \tag{45}$$

Substituting (45) and (43) into (34) gives

$$d_z = \frac{c(d)}{2c(z')}. \tag{46}$$

Finally, we may substitute the partial derivatives from (46), (45), and (41) into the downward-continuation equation (26). Substituting (41), (45), and (46) into (26) and omitting $M_{z'z'}$, we get

$$\frac{c(d)}{c(z')} M_{dz'} + M_{xx}\left(\frac{c^d}{2c^d - c^{z'}}\right)^2 + \frac{c(d)^2}{4} \cdot \left(\frac{1}{c(z')^2} - \frac{1}{\bar{c}(x, d, z')^2}\right) M_{dd} = 0. \tag{47}$$

If we assume that the velocity of waves in the earth depends only on depth, then we may take $c(z') = c(x, d, z')$ and (47) reduces to

$$M_{dz'} = -\frac{c(z')}{c(d)}\left(\frac{c^d}{2c^d - c^{z'}}\right)^2 M_{xx}. \tag{48}$$

This equation has the same mathematical form as (9) and the same computer algorithm may be used again.

Equation (48) is limited to rays of moderate angle from the z axis. This equation was used for Figures 11, 13, 14, 18, 19, 20, and 21. In the remaining figures where steeper ray angles were of interest, the M_{zz} term was included by the method of Claerbout (1971a).

SHOT-GEOPHONE OFFSET

If first-order terms in the offset parameter x/d had been retained in the deduction of (27), we would have instead

$$\left(\partial_d + \frac{x}{d}\partial_x\right)\partial_{z'} M = -\left(\frac{d}{2d - z'}\right)^2 \partial_{xx} M. \tag{49}$$

The new term in (49) becomes important if the inequality

$$1 \gg \frac{x}{d}\frac{M_x}{M_d} \tag{50}$$

is not sufficiently strong.

The parameter x/d is a measure of shot-geophone offset. The expression M_x/M_d is the tangent of the local angle of dip. Thus, the new term in (49) is not really necessary if the offset, or the dip, or the product of offset with dip is sufficiently small. Thus, we may expect that the simpler equation without the offset term will be valid for migrating data over a fairly large range of offsets if the dip is not too large. Now consider the occurrence of the symbol x in the coefficient x/d and as an argument of $M(x, d, z')$. On the basis of our derivation, these two x's are the same x. However, since x/d can usually be neglected (i.e., reset to an arbitrary small value), we may regard the x in x/d as independent of the x in $M(x, d, z')$. In $M(x, d, z')$ the argument x refers to the horizontal coordinate (depth point) on the depth profile. In x/d the x refers to the offset of the shot from the depth point. We have shown that this offset is not very important. This is a mathematical justification for stacking moveout-corrected data from many shotpoints before migration in cases where the product of refractor dip and offset is small. Problems may be expected to arise when the dip or offset is great enough. This may correspond to Levin's statement (1971), based on ray geometry, that stacking velocity depends on dip. A procedure (refer to Figure 5) will be described which is intended for use with data which has several offsets or dips (hence several stacking velocities) present in the same region of y and t.

First, moveout-corrected time sections S_0, S_1, S_2, $\cdots$ can be constructed such that S_0 is made up of all data of zero and small offset S_1 is data with a somewhat larger offset, etc. Then the different S_j may be separately migrated through the use of (49) or its generalization to depth-variable velocity. Since the earth itself is invariant to changes in shot-receiver offset, the migrated S_j should also be invariant to offset. Thus, regardless of dip, the migrated S_j should stack without destructive interference. In fact, the velocity which should be used in construction of the S_j is that for which the migrated S_j stack best.

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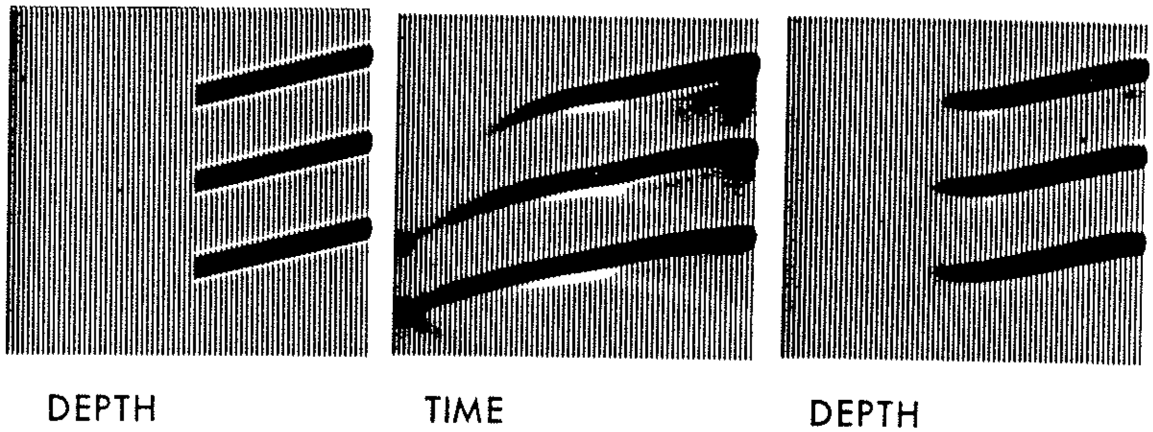

FIG. 12. Like Figure 11 except that the beds have dip. Note that the hyperbolas stay in place but in the time data the interfaces slide down and to the left. (They migrate.) The hyperbolas in the time data would become far more dominant if data were recorded with automatic gain control.

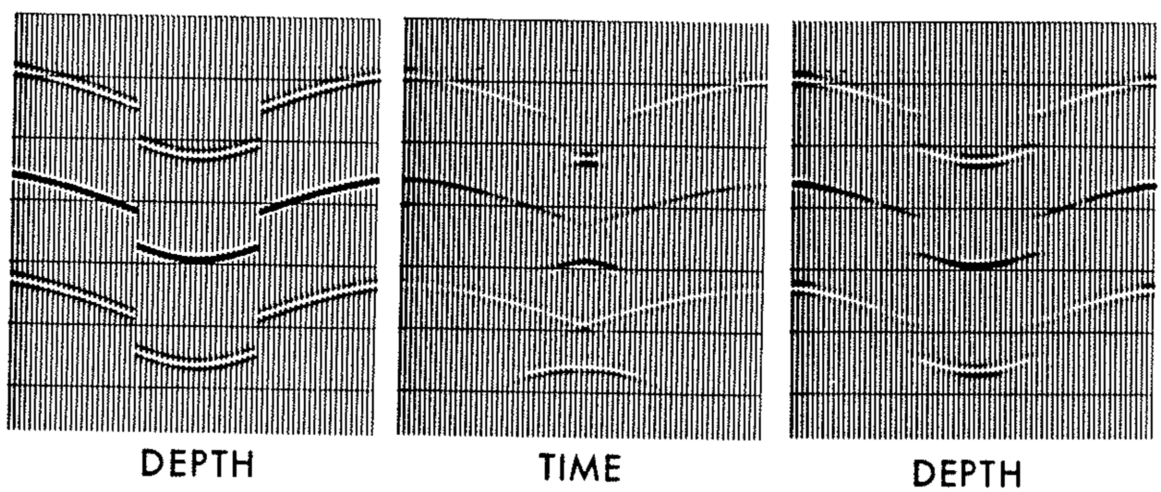

FIG. 13. The classical graben or buried-channel model. Each discontinuity in the depth section gives rise to a faint hyperbola in time.

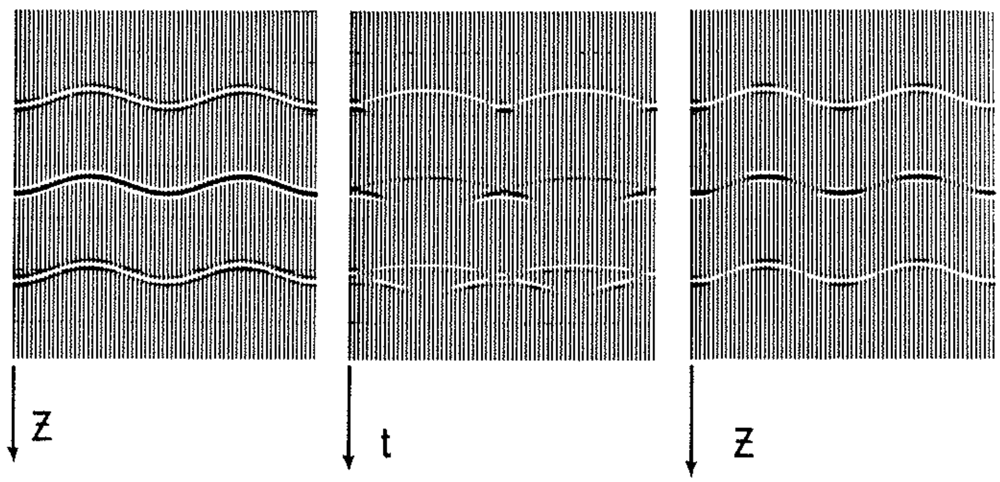

FIG. 14. Sinusoidal interfaces causing many buried foci. Traveltime triplication may be observed readily at the buried foci. Amplitude variations along the reconstructed interfaces arise because dip filtering has been applied on both the data-construction step and the model-reconstruction step.

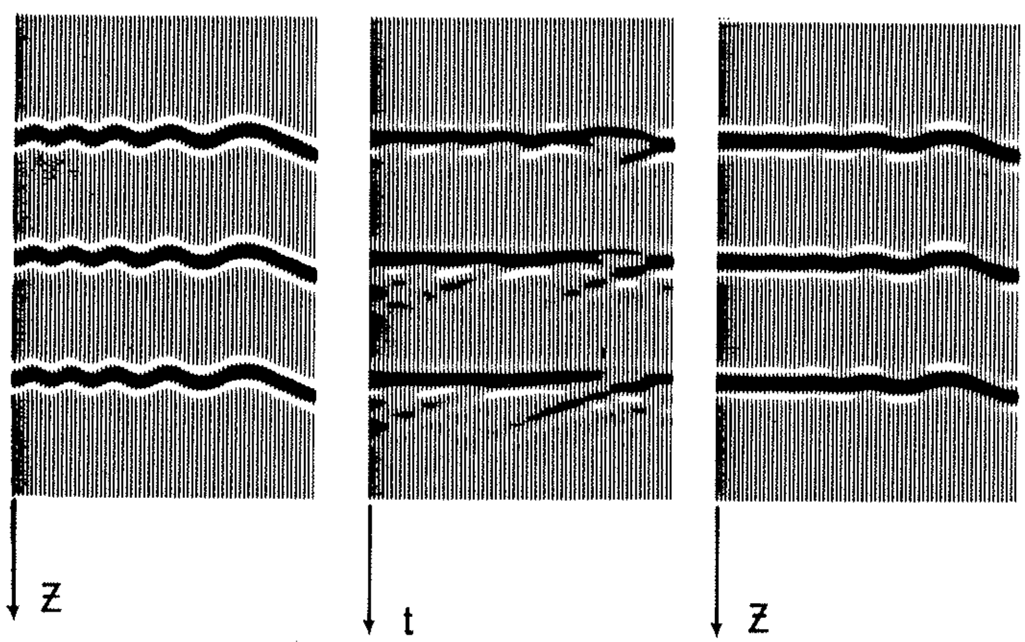

FIG. 15. Undulating interfaces illustrating that horizontal resolution is equal to or less than vertical resolution. A horizontally propagating wave can give as good a horizontal resolution as a vertically propagating wave gives vertical resolution. The dip-filtering operations mentioned in Figure 6 will tend to further degrade horizontal resolution. It will be noted that the rapid oscillations on the left side of the interfaces have not been reconstructed in the rightmost frame.

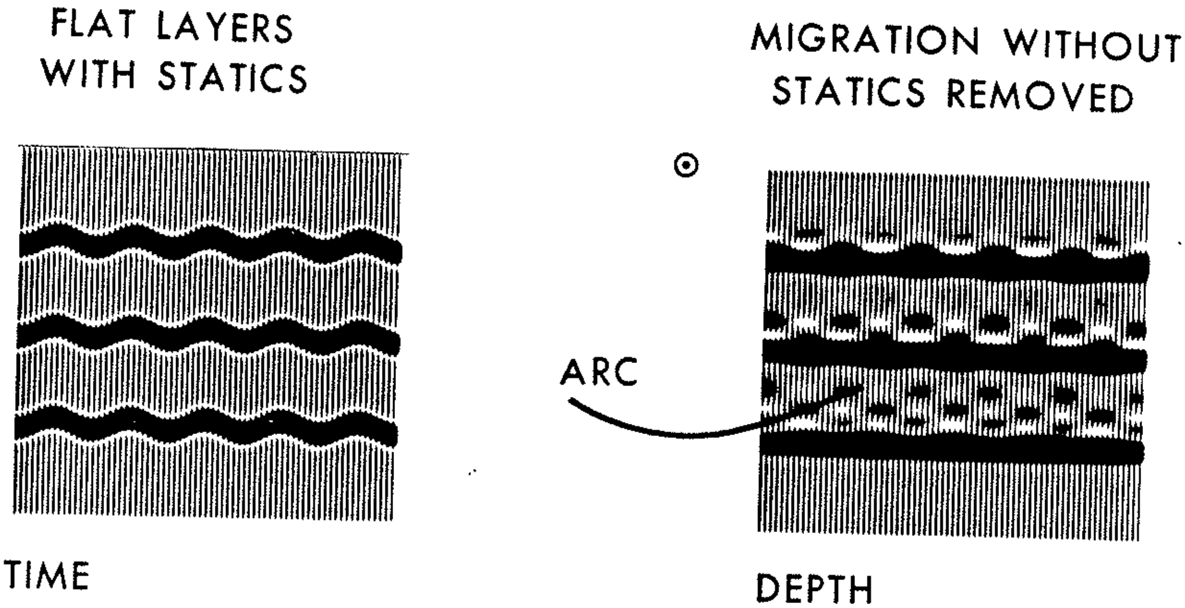

FIG. 16. The effect of unremoved static errors on migration. The left frame depicts reflections from flat layers plus a static timing error which varies sinusoidally from trace to trace. The result of migration is shown on the right. Note that maxima may be aligned into circular arcs.

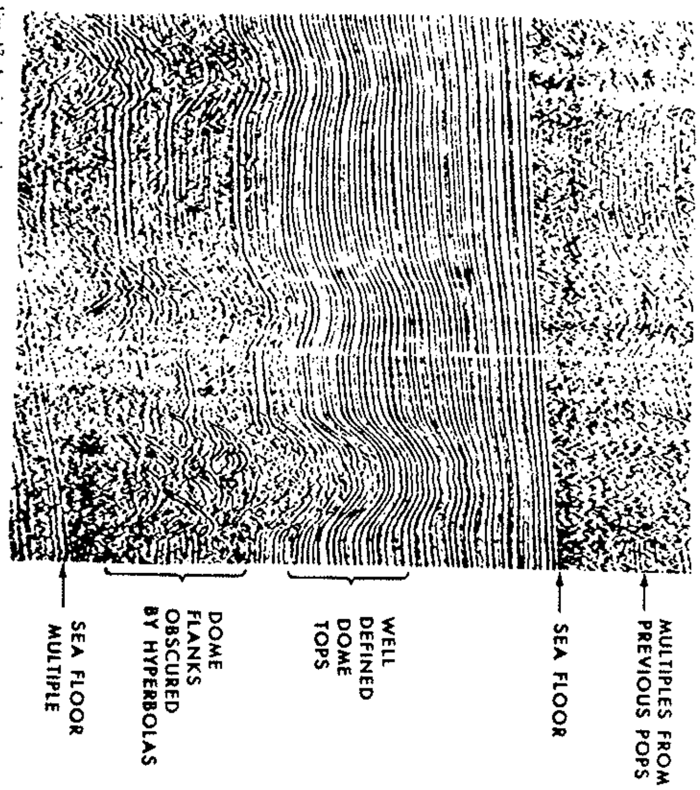

FIG. 17. A seismic section across some diapir like structures. This unprocessed data is from the Umnak Plateau in the southeastern Bering Sea (Scholl and Marlow, 1970). The water depth is about a mile. The vertical exaggeration is about 8. This is single-channel, constant-offset data. For processing purposes it was considered to be a zero-offset section.

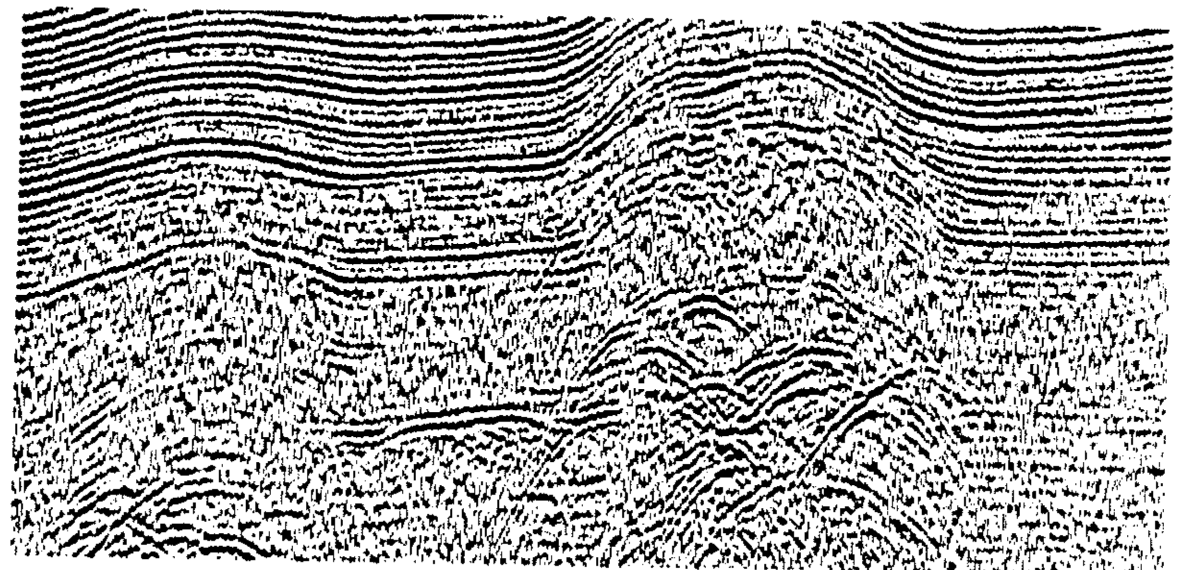

FIG. 18. Enlarged reproduction of the rightmost two domes of Figure 17. Near the top where dips are gentle, the time data may depict a reasonable cross-section of the dome, but the hyperboloids near the bottom arise by scattering from reflecting regions which are considerably smaller than the hyperboloids. Vertical exaggeration is 5.

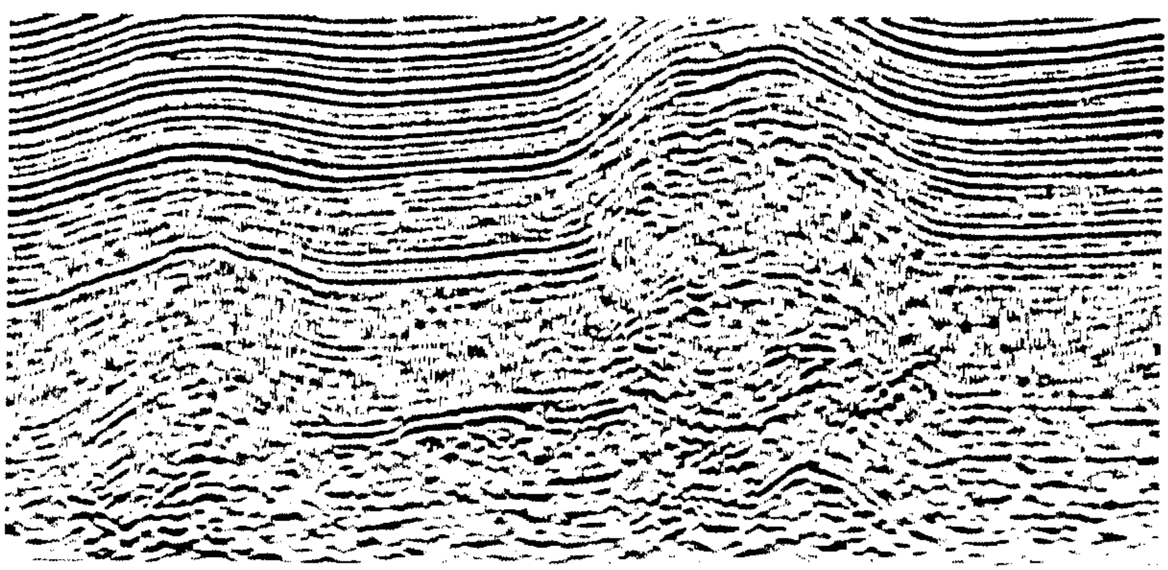

FIG. 19. The first sparker data ever to be migrated by utilizing field data as boundary conditions for a partial-differential equation. These are the data of Figure 18. Hyperbolas have collapsed to smaller scattering centers giving a better picture of the inner part of the dome. Fuzz, due perhaps to sea swell, has disappeared because impossibly steep dips have been attenuated with numerical viscosity. Since all interference effects have not been eliminated (even in the top of the dome) the necessary conclusion is that echoes simultaneously arrive from both sides of the ship. (By the nature of the process, there cannot be simultaneous front-back echoes in the processed data.) Thus, the structure is more complicated than a dome of rotation about the vertical axis. Hence more profiles are required to uniquely delineate the features. The vertical exaggeration is about 5. The data have been processed to dip tangents of about 1/10.

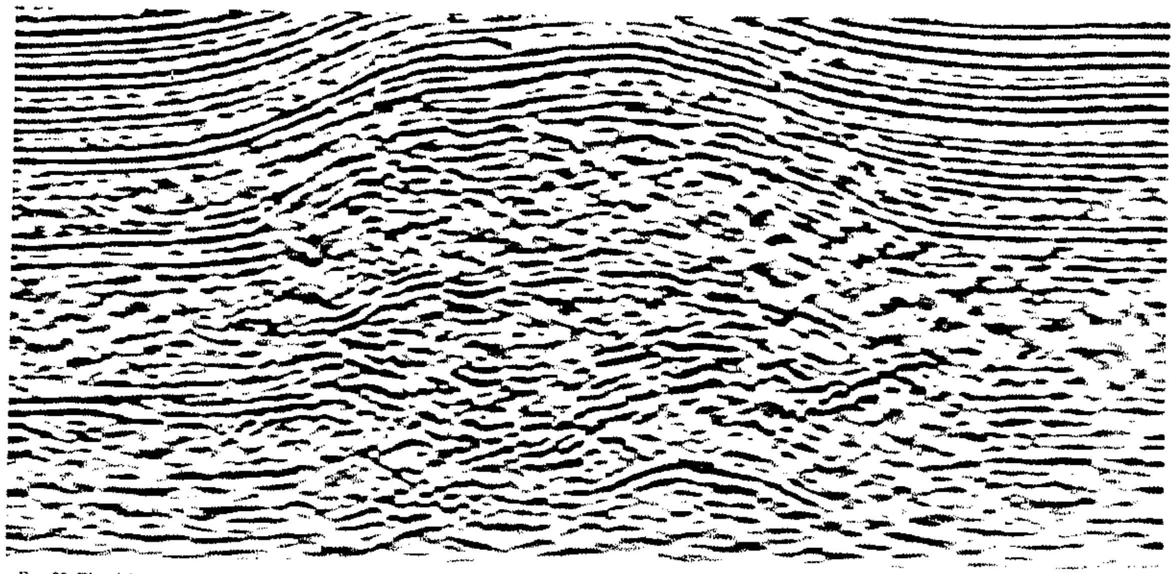

Fig. 20. The rightmost dome of previous figures was further expanded horizontally and processed to dip tangents of about 1/5. Despite the extra processing it is not clear that there has been any improvement over Figure 19. In fact, some circular arcs reminiscent of Figures 7 or 16 are beginning to appear. Such arcs could result from spikes in the data. Actually, we do not believe there are spikes in the data but suspect that these arcs result from some data collection problem as the shipboard AGC changing from channel to channel.

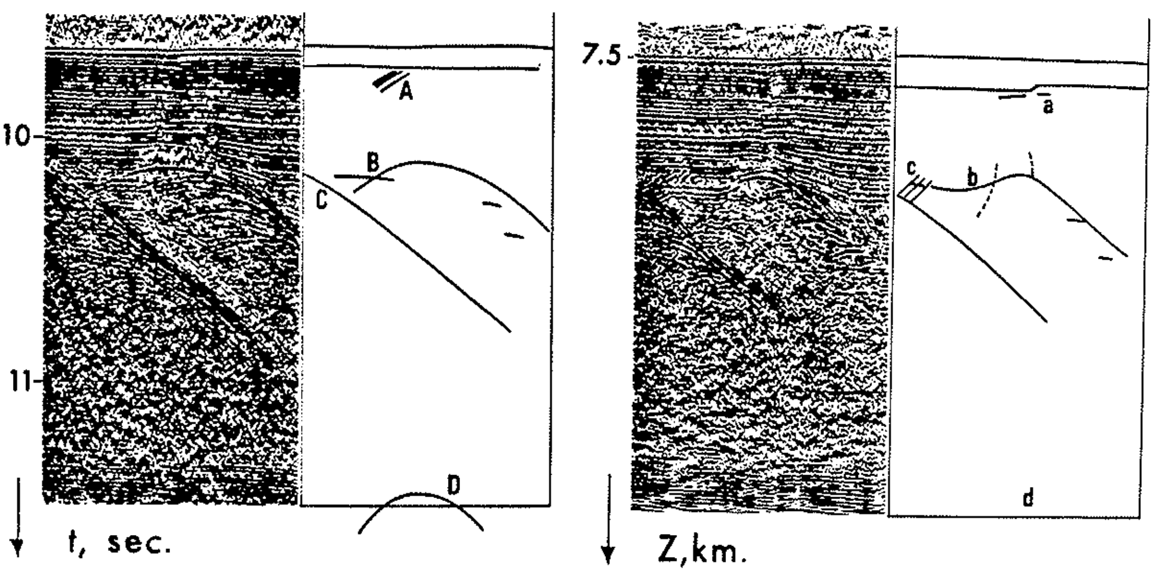

Fig. 21. Migration of Aleutian Trench data. The leftmost frame shows the original data. Note that the top 9 sec, which is the water path, have been omitted. The vertical exaggeration with respect to water velocity is 5.3. Hyperbolic branches at *A* migrate to a small zone *a* where flat-lying sediments are abruptly offset. Traveltime triplication seen at *B* migrates to the concavity at *b* which caused the buried focus. Some undesirable side effects of the process occur around the edges of the frame because of the limited computer memory available. For example the reflector labeled *C* should migrate leftward and upward and partly out of the figure (= the computer memory). In fact, what happens is that a Snell's law of reflection takes place at the side boundary so that the energy appears at *C*. There is also a boundary effect at the bottom of the page. If a hyperbola were to appear as indicated at *D*, its bottom flanks would be cut off by the page boundary. This limits the amount of dip which may be seen at *d*, hence the smoothness in the bottom part of the migrated section.

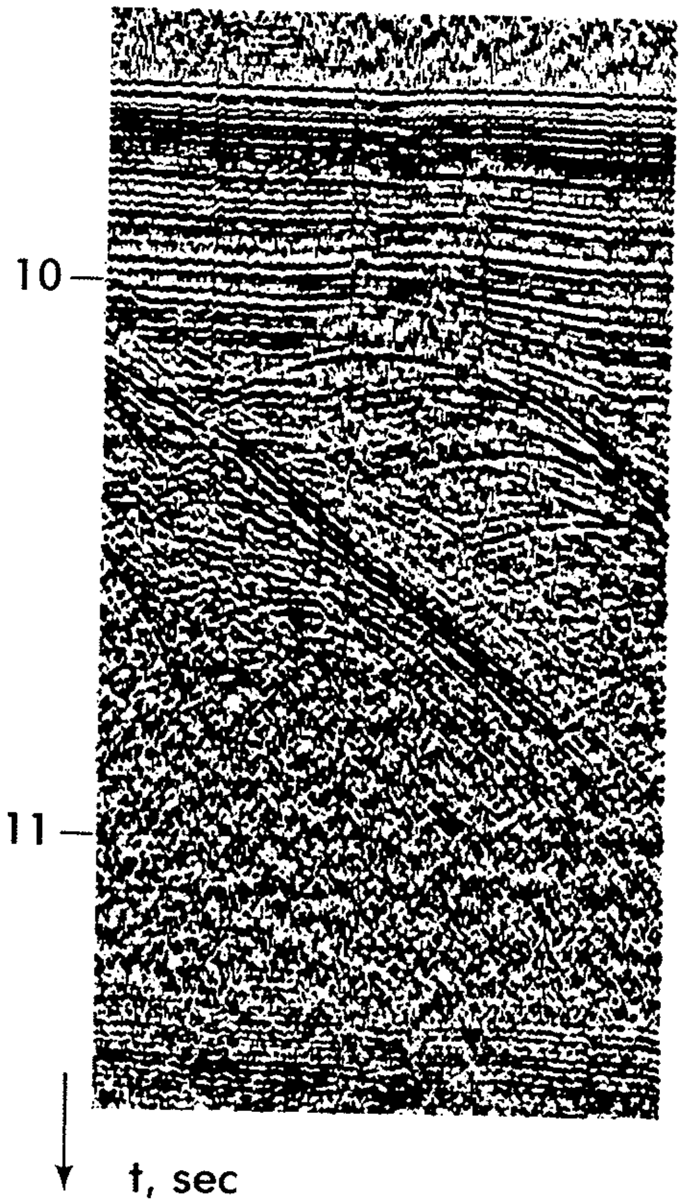

FIG. 22. Enlarged view of the original data shown in Figure 21.

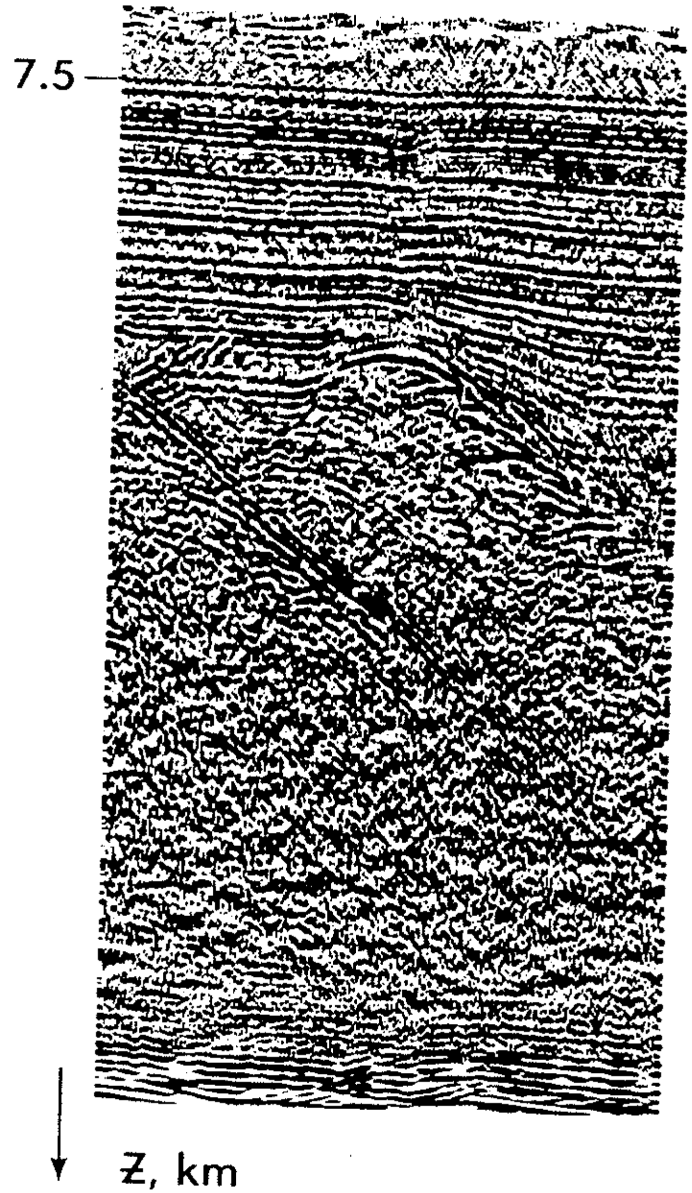

FIG. 23. Enlarged view of the migrated section shown in Figure 21.

This velocity, because it is based on migrated data, should not be subject to the diffraction problem discussed by Dinstel (1971). If the data is "in plane" it should be the true material velocity despite bedding curvature or diffraction.

ACKNOWLEDGMENTS

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